Optical Materials 59 (2016) 91–95

Contents lists available at ScienceDirect

Optical Materials

journal homepage: www.elsevier.com/locate/optmat

Observed light yield of scintillation pixels: Extending the two-ray model

Igor Kantorski, Jacek Jurkowski, Winicjusz Drozdowski *

Institute of Physics, Faculty of Physics, Astronomy and Informatics, Nicolaus Copernicus University, Grudziadzka 5, 87-100 Torun, Poland

ARTICLE INFO

Article history:
Received 16 October 2015
Received in revised form 14 January 2016
Accepted 15 January 2016
Available online 21 January 2016

Keywords:
Scintillator
Scintillation yield
Light propagation
Absorption
Reflection

ABSTRACT

In this paper we propose an extended, two dimensional model describing the propagation of scintillation photons inside a cuboid crystal until they reach a PMT window. In the simplest approach the model considers two main reasons for light losses: standard absorption obeying the classical Lambert–Beer law and non-ideal reflectivity of the "mummy" covering formed by several layers of Teflon tape wrapping the sample. Results of the model calculations are juxtaposed with experimental data as well as with predictions of an earlier, one dimensional model.

1. Introduction

Looking back in time in material sciences we find the paper of Dujardin et al. [1], in which the dependence of scintillation light yield on crystal geometry was discussed so precisely for the first time. After that, an increasing interest in this field was observed, which was related to the development of optical imaging techniques. More and more efforts were made to study light outputs as a function of sample size [2–7] or to get numerical results from optical path modeling with Monte Carlo methods [8–12]. Despite an obvious complexity of the issue, a simple and smart approach was presented by Wojtowicz et al. [5], who proposed a so-called two-ray (2R) model for a scintillation pixel, i.e. a cuboid crystal, the height of which was much larger than the two other dimensions. Although the model limited the problem to a single dimension, in many cases it used to reproduce the experimental observations reasonably well. In this paper we extend this model on two dimensions (2D). The assumption that a scintillation photon has two degrees of freedom allows us to take into account, besides the obvious absorption losses, the reflection losses, which was not possible in the 2R model. The new 2D model will be employed to fit several sets of data points. The quality of the fits will be compared with those obtained within the 2R approach.

2. Assumptions

Recalling the basic ideas of the 2R model, one has to take a straight line representing the one-dimensional crystal, to place the radioactive source at one edge and the photomultiplier (PMT) window at the second edge, and to presume that a luminescence event has a uniform probability distribution in the whole area of the linear crystal. An assumption of a lossless photon reflection from the crystal edge opposite to the PMT is also necessary (in practice, this condition is asserted by a Teflon covering, which usually forms a tight "mummy" over the crystal, leaving the bottom wall uncovered to let the light reach the PMT). The effect of light absorption is considered according to the classic Lambert–Beer law, which leads to the following equation [5] (see Fig. 1 for designations):

$$dLY = LY_0 \frac{1}{2H}\left(e^{-\mu y} + e^{-\mu(2H-y)}\right)dy \tag{1}$$

where μ is an absorption coefficient and LY_0 is a so-called intrinsic light yield, which would theoretically be displayed by a point-size crystal. One can argue that it is not a Cauchy issue and thus it does not have a unique solution. Due to the lack of initial conditions we present an interpretation of the problem, in case of which we do not have to deal with differential equations. Instead of a differential description, we simply take a mean derived with the free parameters, i.e. the initial position and direction of the photon. Let us denote with r the length of the photon path starting from a certain point (of its creation) and reaching the PMT window. Then we have:

$$LY = \langle LY_0 e^{-\mu r}\rangle = LY_0 \frac{1}{2H}\sum_{i=1}^{2}\int_0^H e^{-\mu r_i(y)}dy \tag{2}$$

where i labels the two possible initial directions of the photon (up and down), hence the ½ factor simply normalizes the sum. The ini-

* Corresponding author.
E-mail address: wind@fizyka.umk.pl (W. Drozdowski).

http://dx.doi.org/10.1016/j.optmat.2016.01.033
0925-3467/

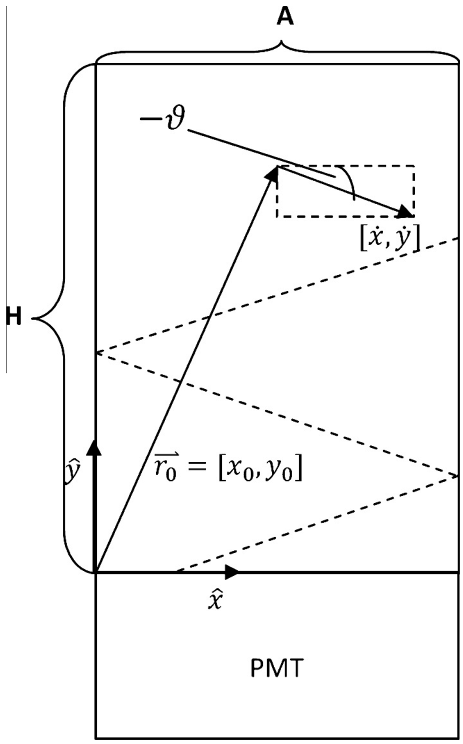

Fig. 1. Simple visualization of the studied system (a scintillation event takes place in the point determined by $\vec{r}_0$ and the photon is propagating in direction described by ϑ; the angle is measured like in an ordinary polar coordinate system).

tial position y is a continuous variable, with its range covering the entire area of a crystal $[0, H]$. For clarity we stress that the notion of a mean is valid if possible initial directions and positions are equally probable. The used notation is in a full agreement with that in [5].

Now, based on the 2R model, we define the observed light yield in the 2D case as a mean over free parameters. However, contrary to 2R, the photon direction is determined by a single continuous parameter ϑ (an angle) and the photon position by two parameters: x_0 and y_0 (Fig. 1). We suppose that the photon path length r depends on these three parameters, i.e. $r = r(\vartheta, x_0, y_0)$. As mentioned before, not only light absorption is taken into account, but reflection losses as well. To describe the latter we use a constant reflection coefficient R, which quantifies how much light remains inside the crystal after a single act of reflection from the Teflon covering the wall ($I = I_0 R$). It can easily be imagined that the contribution from a single photon to LY will be reduced by a factor of R raised to the power equal to the number of reflections that this photon has experienced until reaching the PMT window. Let us to denote this number of reflections with o and assume that it depends on each of the free parameters, i.e. $o = o(\vartheta, x_0, y_0)$, like in case of r. The mean value can be now written as:

$$LY = \langle LY_0 R^{o(x_0,y_0,\vartheta)} e^{-\alpha r(x_0,y_0,\vartheta)} \rangle = LY_0 \frac{1}{2\pi AH} \int_0^{2\pi} \int_0^{H} \int_0^{A} R^{o(x_0,y_0,\vartheta)} e^{-\alpha r(x_0,y_0,\vartheta)} dx_0 dy_0 d\vartheta \tag{3}$$

and we have to aim at finding the expressions for o and r.

3. Number of reflections

We firstly note that the solution hardly depends on the initial direction of the photon propagation. When $\vartheta \in [0, \pi)$, the scintillation photon is passing upward and before reaching the PMT window it has to be reflected from the upper edge of the crystal. In turn, if $\vartheta \in [\pi, 2\pi)$, the photon is passing downward and it reaches the PMT window without changing its vertical velocity component.

Considering $\vartheta \in [\pi, 2\pi)$ we claim that the number of reflections of the photon before reaching the PMT window is equal to:

$$o = \left\lfloor \frac{y_0 - \chi(\vartheta)|\tan\vartheta|}{A|\tan\vartheta|} \right\rfloor + 1 \tag{4}$$

where

$$\chi(\vartheta) = \begin{cases} A - x_0, & \vartheta \in \left[-\frac{1}{2}\pi, \frac{1}{2}\pi\right) \\ x_0, & \vartheta \in \left[\frac{1}{2}\pi, \frac{3}{2}\pi\right) \end{cases}$$

$$\lfloor x \rfloor = \max_{k \leqslant x}\{k : k \in Z\} \quad \text{(floor function)}$$

The function $\chi(\vartheta)$ measures the distance in horizontal direction between the initial position of the scintillation photon and the wall pointed by its initial direction. Introducing an extra variable $n = o - 1$ we notice that $n = l/(A|\tan\vartheta|)$, where l is the distance in vertical direction between the points of its first and last reflection before reaching the PMT window, and $A|\tan\vartheta|$ is the vertical distance between two subsequent reflections (Fig. 2). Hence n counts the photon reflections, reduced by one. Let us divide the vertical distance between the photon initial position and the PMT window into three parts:

$$y_0 = y_i + l + y_f \tag{5}$$

where y_i represents the distance between the vertical coordinates of the initial position and the first reflection point, and y_f is the distance between the vertical coordinates of the last reflection point and the PMT window (Fig. 2). Due to unknown position of the last reflection y_f remains undetermined at the moment. However, if we knew the horizontal distance between the initial position and the wall of the first reflection, we could determine y_i. Therefore we use the already introduced $\chi(\vartheta)$ function and get $y_i = \chi(\vartheta)|\tan\vartheta|$.

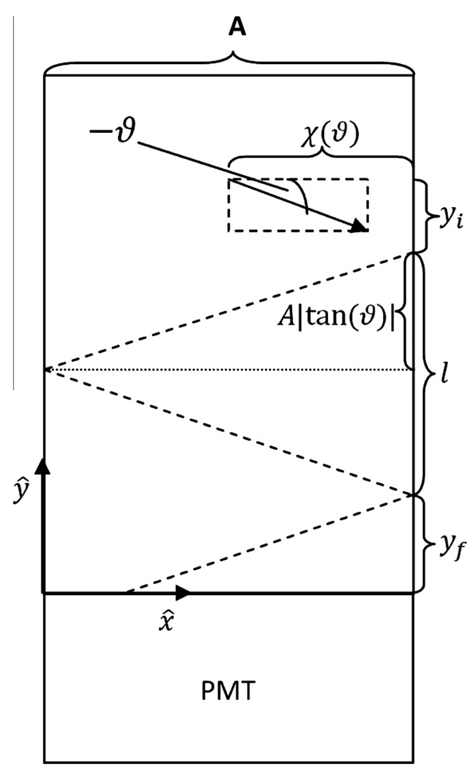

Fig. 2. Division of the vertical distance into three parts.