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Theory of the propagation dynamics of spiral edges of diffusion flames in von Kármán swirling flows

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ABSTRACT

This analysis addresses the propagation of spiral edge flames found in von Kármán swirling flows induced in rotating porous-disk burners. In this configuration, a porous disk is spun at a constant angular velocity in an otherwise quiescent oxidizing atmosphere. Gaseous methane is injected through the disk pores and burns in a flat diffusion flame adjacent to the disk. Among other flame patterns experimentally found, a stable, rotating spiral flame is observed for sufficiently large rotation velocities and small fuel flow rates as a result of partial extinction of the underlying diffusion flame. The tip of the spiral can undergo a steady rotation for sufficiently large rotational velocities or small fuel flow rates, whereas a meandering tip in an epicycloidal trajectory is observed for smaller rotational velocities and larger fuel flow rates. A formulation of this problem is presented in the equidiffusional and thermodiffusive limits within the framework of one-step chemistry with large activation energies. Edge-flame propagation regimes are obtained by scaling analyses of the conservation equations and exemplified by numerical simulations of straight two-dimensional edge flames near a cold porous wall, for which lateral heat losses to the disk and large strains induce extinction of the trailing diffusion flame but are relatively unimportant in the front region, consistent with the existence of the cooling tail found in the experiments. The propagation dynamics of a steadily rotating spiral edge is studied in the large-core limit, for which the characteristic Markstein length is much smaller than the distance from the center at which the spiral tip is anchored. An asymptotic description of the edge tangential structure is obtained, spiral edge shapes are calculated, and an expression is found that relates the spiral rotational velocity to the rest of the parameters. A quasiestatic stability analysis of the edge shows that the edge curvature at extinction in the tip region is responsible for the stable tip anchoring at the core radius. Finally, experimental results are analyzed, and theoretical predictions are tested.

1. Introduction

Swirling flows enhance fuel and oxidizer mixing at the molecular level, promote combustion and flame propagation [1,2] and reduce emissions [3]. Examples of combustion enhancement by increased mixing occur naturally in fire whirls, or in specific engineering designs such as swirl combustors in gas turbines, swirl-generating inlet ports and swirl piston bowls in internal combustion engines. The interaction of flames with swirling air flows in combustion chambers can enhance local extinction for sufficiently large strain rates or heat losses. The boundaries produced by these local extinction phenomena are edge flames, which propagate through the mixture with characteristics similar to those of deflagrations [4]. In swirling boundary layers, local quenching can produce a number of flame patterns and flame fronts, such as straight edge flames, single spiral edge flames, multiple spiral edge flames, flame rings and flame holes, that propagate in the mixture in a nontrivial manner [5–8]. This paper addresses the dynamics and structure of the spiral edge flames found in earlier experiments [5–7]. A snapshot of this type of flame pattern is shown in Fig. 1.

From a broader physical standpoint, spiral patterns are ubiquitous in nature. The Belousov–Zabothinsky reaction [10] and the catalytic surface oxidation of CO [11] are examples of spiral pattern formation found in physical chemistry. Cell aggregation [12] and calcium waves [13] are examples of spiral patterning in cell signaling, and cardiac fibrillation waves take the form of meandering spirals. However similar at first sight, pattern formation in diffusion flames is a qualitatively and quantitatively different problem than those cited above, which mainly correspond to reactive–diffusive systems [10–13]. Spiral diffusion flames display three-dimensional diffusion and advection effects that make their analytical and

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Fig. 1. Spiral edge flame (adapted from [9]).

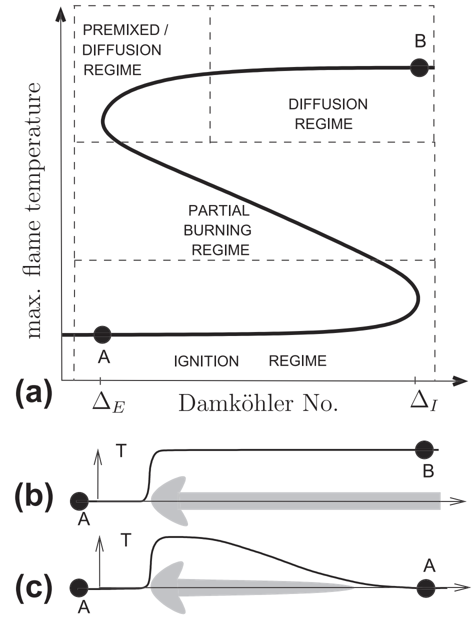

Fig. 2. (a) Schematic representation of the S-curve response of a diffusion flame. The frozen and Burke–Schumann rest states are denoted by A and B, respectively. (b) Temperature profile along the stoichiometric line of a vigorously burning edge flame. (c) Temperature profile along the stoichiometric line of an edge whose diffusion flame has undergone extinction.

numerical tractability quite more challenging. Additionally, the strong non-linearities associated with the reaction term in flames are typically exponential in the temperature as in the Zeldovich–Frank–Kamenetskii theory [14], which produces sharp interfaces and singular behaviors, whereas classical developments in the reactive–diffusive systems of [10–13] have typically used smoother polynomial non-linearities to account for the propagator source as in the Kolmogorov–Petrovskii–Piskunov theory [15]. Finally, unlike earlier analyses performed on the modelling the physico-chemical processes [10–13], the conservation equations and chemical reaction rates used in combustion systems stem from fundamental principles of the kinetic theory of gases, which confers more rigor on the present investigation. Despite all these differences, excitability is a common characteristic shared by all these systems. The concept of excitability of a system regards its ability to trigger abrupt and substantial responses –by means of the autocatalytic production of a propagator or trigger variable– to disturbances from a rest state that cross certain characteristic thresholds [16]. After such a response the system momentarily shows a refractory behavior moderated by a controller or refractory variable, in that the system is immune to further stimulation and eventually recovers full excitability.

In the context of flame propagation in stratified mixtures, excitability is related to the reactivity of the mixture and the high sensitivity to temperature because of the effectively high overall activation energy involved in typical combustion chemical reactions. Although the trajectory of a fluid particle occurs generally in a multidimensional space, its initial and final states can be placed on a S-curve response of a diffusion flame [19] as shown in Fig. 2a, which represents the maximum temperature as a function of the reduced Damköhler number Δ, which is defined as

$$\Delta = \frac{t_d}{t_c}, \tag{1}$$

where t_d is the diffusion time through the flame or flame-transit time, and t_c is the local chemical time in the flame.

Spatial regions of sufficiently low temperature, in which the mixture remains chemically frozen, may be thought of as a rest state, from which an appreciable excursion may occur when Δ is sufficiently large to trigger thermal runaway and ignite the mixture. Flame propagation and diffusion of heat into neighboring regions occur once the mixture has been ignited, causing the excitation process to spread spatially into zones initially frozen. For large activation energies and adiabatic systems, the region downstream from the front is close to a Burke–Schumann or equilibrium rest state, involving a reaction zone into which each reactant diffuses and reacts producing a diffusion flame, as in Fig. 2b. This trailing flame extends infinitely far downstream from the front as in an adiabatic, equidiffusive edge flame or triple flame [4,20–24].

For a spiral edge flame, the system achieves the rest state again by means of an extinction tail, as in Fig. 2c. As shown further below, lateral heat losses to the burner surface extinguish the flame when the flame temperature is sufficiently small for the local Δ to be small, such that the local chemical time becomes large and of the same order as the diffusion time, which produces leakage of unburnt reactants, an associated flame-temperature drop and flame extinction. After extinction, the gaseous mixture is at first refractory to another disturbance, but it finally recovers full ignitability when the reactants are replenished by advection, reproducing the same edge-flame pulse after one revolution provided that no mixing hysteresis has taken place. While this discussion describes how spiral-flame phenomena may be related in general to spiral waves in excitable media, it will be seen later that Fig. 2a does not precisely describe the specific experiments of Fig. 1, in that the indicated ignition event, in fact, is not involved in the physics of the actual process.

Spiral-like flame patterns of different nature than the ones treated in this study have been previously reported for fully premixed combustion systems. Examples of these phenomena are the pelton-like flames found in the combustion of methane and air in radial microchannels [25], the spiraling instabilities observed on the surface of expanding spherical premixed flames in hydrogen–air mixtures [26], and the flames found in the combustion of lean mixtures of butane and oxygen in a pipe [27]. These patterns occur in premixed systems, and their dynamics can be described by two-dimensional reaction–diffusion conservation equations.

The paper is organized into five additional sections. Section 2 is dedicated to a general formulation of the problem, in both the laboratory and moving reference frames, and within the framework of a thermodiffusive, equidiffusional model using a single-step chemical reaction of large activation energy. The effects of lateral heat losses to the burner wall on a nearly straight edge flame are addressed in Section 3 by performing scaling analyses of the conservation equations and integrating numerically a two-dimensional model problem. The results in this section do not address the influences of curvature but instead identify limits on the magnitudes of the curvature. Section 4 is dedicated to the study of the tangential