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# Analyzing a lost-sale stochastic inventory model with Markov-modulated demands: A simulation-based optimization study

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ABSTRACT

A consumer demand that presents auto-correlated components is a class of demand commonly found in competitive markets in which consumers may develop preferences for certain products which influence their willingness to purchase them again. This behavior may be observed in inventory systems whose products are subject to promotion plans in which mechanisms that incentivize the demand are implemented. Inventory systems that ignore these dependency components may severely impair their performance. This paper analyzes a stochastic inventory model where the control review system is periodic, is categorized as a lost-sale case, and is exposed to this class of auto-correlated demand pattern. The demand for products is characterized as a discrete Markov-modulated demand in which product quantities of the same item may relate to one another according to an empirical probability distribution. A simulation-based optimization that combines simulated annealing, pattern search, and ranking and selection (SAPS&RS) methods to approximate near-optimal solutions to this problem is employed. Lower and upper bounds for a range of near-optimal solutions are determined by the pattern search step enhanced by ranking and selection—indifferent zone. Results indicate that inventory performance significantly declines as the autocorrelation increases and is disregarded.


## 1. Introduction

A supply chain refers to all parts that are systematically involved in fulfilling a customer demand [1]. As such, a supply chain entails the coordination of resources to move goods or provide services from firms to consumers. In this sense, consumer demand is a critical component of the supply chain that has significant implications for the operational and strategic goals of the firm. Thus, understanding the effects of uncertainty on demand is a long-standing interest from practical and theoretical perspectives.

Demand uncertainties may have an impact on supply chain performance, and therefore, it may compromise firm's ability to control costs and make profits. These demand uncertainties may be substantial in some settings. Such demand fluctuations oblige operational and strategic managers to frequently review decisions related to sales and operations planning. Firms that possess the ability to adapt and change to these demand fluctuations are more likely to succeed in highly competitive environments.

Recognizing and addressing issues related to planning and controlling operations subject to uncertain demand may be a differentiator that distinguishes successful supply chain and others. Understanding both relevant sources of uncertainty and their effects is imperative for addressing these issues. This necessarily requires the use of a capable framework that allows managers to process this information and create solutions that minimize the adverse effects on the firm. This information may enable the firm to address these negative effects, and hence, maximize opportunities to better match the supply with the demand.

While uncertain demands continuously change over time, the arrangement of supply chain assets commonly shows high degrees of rigidity. Generating solutions to address fluctuating demand issues in this environment may be challenging. Managers employ supply chain management techniques (e.g., reduce inventory) to effectively manage resources and products to maximize supply chain surpluses and minimize risks [1]. From the supply perspective, inventory and capacity management have been recognized as

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effective tools for balancing the supply and demand. Conversely, from the demand angle, advertising and marketing tools may be used to promote increases in the demand. Furthermore, information, sourcing, and pricing are considered cross-functional levers that enhance supply chain management.

Pricing is a cornerstone for spurring revenue increases that stem from supply chain assets utilization. Clearly, the amount of product required by the uncertain consumer demand is shaped by pricing decisions that invariably leads to revenue generation. Pricing decisions are central in promoting sales, and therefore, in the ability to generate revenues by managers. However, promotion policies are one of the most important contributors of the "bullwhip effect" [2]. Bullwhip effect refers to the increased variance in demand that is observed as we move upstream in a supply chain. Thus, the demand variance shows to be greater in supplier orders than in retailer orders or sales. Supply chain members may be misled in making inventory decisions, and hence, experience significant losses as the information from these orders are inaccurate. However, the use of promotions is prevalent in supply chains as many organizations are inclined to use pricing or quantity discounts to spur revenues while engaging in riskier decisions that may erode prospective profits.

An accurate analysis of the effects of promotion or another type of pricing vehicle to increase the demand over other logistical levers such as inventories may be needed. Anticipating the inventory and capacity policies that support the implementation of such mechanisms is critical in maintaining a firm's competitive edge. Pricing promotion and quantity discount decisions that are frequently practiced in retail environments may be improved from properly considering the effects that such incentives may have on inventory management. In this environment, it is well known that advertising campaigns promote the buying of a product (or combination of products) while receiving another number of items free of charge or at discount. These promotions may create a dependency effect (e.g., induced autocorrelation) on the probabilistic demand that may be determined by analyzing and modeling its stochastic pattern.

Ref. [3] shows that a positive autocorrelation is commonly found among a large number of retail products in varying proportions. However, most inventory models developed in the literature, assume that demand can be described as a continuous function whose observations are identically independently distributed (IID). This assumption may be misleading as the performance of inventory systems that fail to consider dependency components declines as increases in demand variability produces stockouts that quickly compromise acceptable service levels [2,4]. Some authors acknowledge this dependency and formulate demands as serially correlated characterizations (e.g., Miller [5,6], and [3,7]. The use of continuous formulations is largely predominant among the few inventory models that consider auto-correlated demands (e.g., Kurata and Liu [8], and Diaz and Ezell [9] employ Autoregressive AR(1)).

Dependency patterns may be complex and largely dependent on the promotion scheme that affects customer behavior. For example, a consumer requesting a product may additionally request two more identical items if the promotion policy offers a benefit according to a probability distribution. Likewise, for another set of customers, the promotion policy might vary, and hence, induce a different demand patterns for the consumption of the same product. Customer segmentation that demands the same product at different rates has been largely studied in the marketing literature. These different auto-correlated demand patterns for the same product may have important consequences for the inventory. One way that consumer demand with the described autocorrelation patterns can be formulated consists of using a Discrete Markov-modulated Chain (DMC) formulation. Finite quantity demanded can be modeled as discrete states that are connected through transition probabilities that generate auto-correlated dependencies constrained by limits in the quantities demanded by the promotion. The mathematical formulation of inventory systems like this are deemed as intractable due to complicated multivariate integration. Supply chain literature that explores the performance of inventory systems that are subject to auto-correlated demand viewed as DMC that consider multiple probabilities such those induced by promotion schemes is scant.

The purpose of this paper is to characterize and study an inventory system that is conditioned to DMC demand frequently found in competitive markets that use promotions to induce consumption in segmented markets. This work extends the work of Diaz and Ezell [9] as it employs a simulation based-optimization approach to approximate solutions to a lost-sale inventory system in a different stochastic inventory environment. When probabilistic distributions, as the one considered in this paper, are intractable, simulation-based optimization such as Markov Chain Monte Carlo overcomes this limitation by generating a sample sequence where each decision point has the desired distribution [10]. This study generates policy solutions to inventory systems whose demand contains dependent components that can be described as DMC. Furthermore, the main effects and interactions over holding, ordering, and stockout costs that define the performance of these systems are investigated through statistical analysis as the auto-correlated demand increases.

The stochastic inventory problem involves a single-item whose replenishment takes place over the next business day and not as a perpetual inventory review policy. The analyzed inventory control system assumes a ($s$, $S$, $R$) periodic review policy where $s$ is reorder point, $S$ is the targeted inventory level and $R$ is the reviewing period. The simulation optimization method employed to analyze this inventory management problem is based on a technique that combines simulated annealing (SA) approach with pattern search (PS) and ranking and selection (R&S). This method estimates solutions to the objective function that randomly generate a location in the feasible space and apply randomized (SA) and deterministic (PS and R&S) rules to select whether to move to a new location on the path to a solution. As the auto-correlated component increases, the R&S component play a more relevant role since more replications are needed to assess prescribed levels of user-defined Indifference Zone (IZ). We now offer a brief review of the three procedures used in this study.

Simulated annealing is a probabilistic method proposed in Kirkpatrick et al. [11] and Cerny [12] for finding the global minimum of a cost function, in a large search space, that may possess several local minima. It is often used when the search space is discrete. It works by emulating the physical process whereby a solid is slowly cooled so that when eventually its structure is frozen, this happens at a minimum energy configuration. The notion of slow cooling is implemented in the simulated annealing algorithm as a slow decrease in the probability of accepting worse solutions as it explores the solution space.

Pattern search is a family of numerical optimization methods that do not require the gradient of the problem to be optimized. Hence PS can be used on functions that are not continuous or differentiable. The name pattern search, was coined by [13]. An early PS variant is attributed to Fermi and Metropolis as described by Davidon [14] who summarized the algorithm as: Varying one theoretical parameter at a time by steps of the same magnitude and when no such increase or decrease in any one parameter further improved the fit to the experimental data, step sizes are halved and the process is repeated until the steps are deemed sufficiently small.

Ranking and Selection is a group of statistical techniques developed to address the optimization problem associated with the goal of selecting the "best" simulated system configuration (s) from a given solution space, where "best" refers to the maximization or