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Technical Paper

Note on a unified approach to the single-machine scheduling problem with a deterioration effect and convex resource allocation

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ABSTRACT

In many resource allocation problems in physical or economic systems, a linear resource consumption function is commonly considered, and job processing times are assumed to be fixed parameters. However, the former assumption fails to reflect the law of diminishing returns, and the latter may be controlled by changing the allocation of resources to jobs. Motivated by these observations, we provide a unified model for solving single-machine scheduling problems in which each job's processing time is a function of its starting time and convex resource allocation. The objective is to find the optimal sequence of jobs subject to a limited resource consumption. We first show how this unified model can be useful in solving scheduling problems under due date assignment considerations. We analyze the problem with four different due date assignment methods, and our objective function includes costs for earliness, tardiness and due date assignments. We also consider scheduling problems without involving due date assignment decisions. The objective function is to minimize the makespan, total completion time, total absolute variation in completion times, and total absolute variation in waiting times. We show that several existing well-known problems can be reduced to a special case of our unified model and solved in $O(n\log n)$ time.

1. Introduction

A common assumption in classical scheduling problems is that the job processing times are fixed parameters. However, in many realistic situations, job processing times may be controlled by changing the allocation of resources to the jobs, which may result in further efficiencies. Scheduling problems with controllable processing times have been studied extensively by researchers (see, for example, Koulamas et al. [13], Leyvand et al. [16], Oron [19], Shabtay [24], Shabtay and Steiner [25], Shabtay and Steiner [26], Yin et al. [33] and so on). Most of the abovementioned studies of scheduling with controllable processing times have focused on two types of models: the linear resource function model and the convex resource function model [25]. In the linear resource function model, it is assumed that the job processing time is a bounded linear function of the amount of resources allocated to the processing of the job, i.e., the *resource consumption function* is of the form

$$p_j(u_j) = \bar{p}_j - a_j u_j,\ 0 \le u_j \le \bar{u}_j \le \bar{p}_j/a_j \quad (1)$$

where $\bar{p}_j$ is the non-compressed processing time for job J_j and the variable $\bar{u}_j$ denotes the upper bound on the amount of resources that can be allocated to job J_j, and a_j is its positive compression rate. However, for many resource allocation problems in physical or economic systems, a linear resource consumption function has only a limited application because it fails to reflect the law of *diminishing returns*. This law states that productivity increases at a decreasing rate with the amount of resources employed. One class of models reflecting this law uses a convex resource consumption function described by the following equation

$$p_j(u_j) = \left(\frac{w_j}{u_j}\right)^k \quad (2)$$

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where $u_j > 0$ is the amount of resources allocated to job J_j, w_j is a positive parameter representing the workload of job J_j, and k is a positive constant.

Scheduling with deteriorating jobs was introduced by Browne and Yechiali [4] and Gupta and Gupta [10]. Since then, machine scheduling with deteriorating jobs has received increasing attention (see, for example, Alidaee and Womer [2], Bachman et al.[5], Cheng et al.[6], Cheng et al.[8], Lai and Lee [15], Pei et al.[21,22], Wang et al [27], Wu et al. [31], Yang and Kuo [32], Yin and Xu [34], Yin et al. [35–39], and Gawiejnowicz [14] for recent state-of-the-art research in this area). Most of the research has focused on a model where the job-independent deterioration rates are identical for all jobs, which is a very realistic setting. For example, Cheng et al. [7] discussed the case of a single-machine and linear job-independent deterioration. The goal is to determine an optimal combination of the due date and schedule to minimize the sum of the due date, earliness and tardiness penalties, particularly in the case of scheduling with deteriorating machines, when all processing times increase by a common factor that is caused by the machines. In this model, they assumed that the actual job processing time is given by the following relationship:

$$p_j = a_j + bS_j \quad (3)$$

where a_j is the *basic processing time* of job J_j, b is the job-independent *deterioration rate*, and S_j is the job *start time*. The authors developed an $O(n\log n)$ time algorithm to solve the problem.

There are also several studies on scheduling with deteriorating jobs and controllable processing times. Wei et al. [30] considered single-machine scheduling with resource-dependent processing times and deteriorating jobs. The objective is to minimize a cost function of the makespan, total completion time, total absolute differences in completion times, and total resource consumption cost. They used polynomial time algorithms in a linear resource function model. Wang and Wang [28] presented polynomial time algorithms for single-machine scheduling problems with convex resource-dependent processing times and deteriorating jobs. The objective function is to minimize a cost function of the makespan, total completion time, total absolute differences in completion times, and total resource consumption costs. Wang and Wang [29] studied a single-machine scheduling problem with due date assignment, in which the processing time of a job is a function of its starting time and resource allocation. They analyzed the problem with two different processing time functions and three different due date assignment methods. The goal is to minimize an integrated objective function, which includes the earliness, tardiness, due date assignment, and total resource consumption costs. For each combination of the due date assignment method and processing time function, a polynomial time algorithm is given. Both [28] and [29] assume that the total amount of resources is large enough, but in practical problems, the total amount of resources may be limited. *A real application of this proposed model lies in dealing with accidents that involve pollution or contamination. For example, one can consider accidents involving oil tankers, where the effort to contain pollution is dependent on the reaction time of the authorities. Furthermore, fixing the vessel and stopping the spill of oil is a function of the resources allocated to this mission. This is also the case when considering the spread of disease; the time required to deal with the disease source is dependent on the resources allocated to that task, whereas dealing with the affected population is dependent on the time required to detect the disease and treat those affected* [19]. Oron [19] considered a single-machine scheduling model with general linear deterioration and convex resource functions. They presented a polynomial time solution for minimizing the makespan. For the total flowtime criterion, they computed the optimal resource allocation policy for a given job instance and showed that the problem is at least as hard as the case with non-controllable jobs.

Table 1
The list of notations.

p_j	Processing time of job J_j
w_j	Workload time of job J_j
d_j	Due date of job J_j
S_j	Start time of job J_j
C_j	Completion time of job J_j
$E_j = max\{0, d_j - C_j\}$	Earliness of job J_j
$T_j = max\{0, C_j - d_j\}$	Tardiness of job J_j
u_j	Amount of resources allocated to job J_j
U	Total amount of non-renewable resources
k	Positive constant

In this paper, we provide a unified model for solving single-machine scheduling problems with a deterioration effect and convex resource allocation. It is assumed that the processing time of a job is a linear function of its start time and that the actual job processing times are controlled by a convex resource allocation function.

2. Problem formulation

The problem under investigation can be described as follows.

There are n jobs $J_1, J_2, \ldots, J_n$ to be processed on a single machine. All jobs are available at time zero, and no preemption is allowed. The actual processing time of job J_j ($j = 1, \ldots, n$) consists of two terms, a resource-dependent term and a time-dependent term, which are given as follows:

$$p_j = \left(\frac{w_j}{u_j}\right)^k + bS_j \quad (4)$$

where p_j is the actual processing time of job J_j if allocated u_j units of a non-renewable resource and if its processing begins at time S_j. Let b be a non-negative parameter reflecting the deterioration rate of jobs, and let k be a positive parameter that defines the structural form of the resource allocation function. Let U be the total available amount of resources. Thus, $\sum_{j=1}^{n} u_j \le U$ because one cannot exceed the available amount of resources. The notations used throughout the paper is described in Table 1.

We consider the general problem with a deterioration effect and convex resource allocation, denoted as

$$1|p_j = \left(\frac{w_j}{u_j}\right)^k + bS_j, \sum u_j \le U|f \quad (5)$$

For a given problem, we seek an optimal job schedule combined with an optimal resource allocation policy to minimize the scheduling criterion f.

3. The unified model for single-machine scheduling

In this section, we provide a unified model for solving the problem (5).

We first consider the following optimization problem (P).

$$\begin{aligned} \min \quad f(x_1, x_2, \ldots, x_n) &= \frac{a_1}{x_1^k} + \frac{a_2}{x_2^k} + \ldots + \frac{a_n}{x_n^k} \\ s.t \quad & x_1 + x_2 + \ldots + x_n \le U. \\ & x_j > 0, \quad j = 1, 2, \ldots, n. \end{aligned} \quad (6)$$

where a_j ($j = 1, \ldots, n$), U and k are positive parameters.