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Technical Paper

An inventory model with truncated exponential replenishment intervals and special sale offer

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ABSTRACT

In this study, we develop an inventory model with stochastic replenishment intervals and special sale offer from a supplier. The replenishment interval is assumed to obey a truncated exponential distribution and shortage is partially backordered. Our goal in this research is to maximize the total profit of cost savings due to special sale offer from supplier. A closed-form solution of the model and its convexity condition is developed. A numerical example with real world data is provided to illustrate the theory.

1. Introduction

In recent literature, many researchers modeled optimal inventory decisions under various conditions. Numerous models considering technology are developed. Among them are models with closed-form solutions. Due to uncertainty in technological development, many scholars developed models for uncertain environments. Stochastic models are more general since deterministic data is regarded as special case when the variance of the input data equal to zero. However, due to transportation delays and other uncertainties, most inventory models have supply time variations. These variations may follow a probability distribution. In real life, the stochastic replenishment interval may be exponential, Gamma or truncated exponential distributions. The differences between lead-time and replenishment interval are as follows: lead time is defined as the time between ordering and receiving of a product, while replenishment interval is the time between two receipts of orders. When the replenishment is repeated, we consider lead-time and replenishment interval in the model is the same. Let T stand for the stochastic length of a replenishment interval. Without loss of generality, one can assume that T varies stochastically between two extremes t_{min} and t_{max}. Many researchers assume exponential distribution for the time between two consecutive replenishments (i.e. $t_{min} = 0$ and $t_{max} = \infty$). However, using a simple exponential distribution is not suitable for the case when $t_{min} = \lambda > 0$. This is because $T = 0$ is meaningless in real world problems. Also, we have found this to be true in practice when analyzing a local pharmaceutical manufacturing company's data. In our observation, there is no random value with simple exponential distribution for the replenishment interval. In other words, in recording the previous data for the time between two consecutive replenishments, no value close to zero is observed. The data are all greater than a minimum threshold λ. Thus, a truncated exponential distribution is found to be the most fitting for the observed data.

However, it is very difficult to derive a closed-form solution when the input data are stochastic. Hence, most stochastic models only consider the expected value of the objective function; and only close to optimal solution is obtained for the numerical example. The model is then re-analyzed periodically in a rolling horizon framework to correct the previous errors (if any) in decision making.

This paper is organized as follows: Section 1 discusses the scope and purpose of this research, Section 2 is the literature review, Section 3 states the assumptions, notations and model formulation, Section 4 is the numerical example, and Section 5 is the concluding remarks and suggestion for future researches.

2. Research motivation and literature review

This study is initiated as a result of observing a pharmaceutical manufacturing company in Iran. In the study, we maximize the

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total cost savings profit due to special sale offer from a supplier. An inventory model is developed with stochastic replenishment intervals and special sale offer from the supplier. The replenishment interval is assumed to obey a truncated exponential distribution and shortage is partially backordered. A closed-form solution of the model and its convexity condition is developed. Further details of the problem are given in the following section.

Many researches have been done in the last decades. Arcelus et al. [2] examined a retailer's response to a vendor's trade promotion, followed by an additional (uncertain) period of time. They developed a general special-sales model under uncertainty to maximize retail profit during the deterministic and the stochastic portions of the special sale period. Then, Arcelus et al. [4] modeled the retailer's profit-maximizing retail promotion strategy during a vendor's trade promotion offer. Ertogral and Rahim [6] considered an inventory planning problem when there were random periodic replenishment intervals. Martınez-Ruiz et al. [13] applied the daily store-level data to consider price promotion effects. Sarker and Al-Kindi [14] developed EOQ models with discounted prices for five different cases. They concluded that the annual gain was linearly related to the discount and the on-hand remnant inventory. Abad [1] considered the buyer's response to a temporary price reduction. His model included transportation costs in their purchase decisions. Arcelus et al. [3] considered a profit-maximizing decision process when a vendor offered a temporary sale price. Their policies were applicable to any probability distribution for uncertain discount period. Other researches such as Gupta and Goel [8], Koulamas [11], Wee and Yu [15], Ghosh [7], Dye and Hsieh [5], and Guria et al. [9] focused on other aspects of the problem. Similar to Arcelus et al. [3], most of the literatures maximized the retailer's profit during a vendor's temporary sale price.

Recently, Karimi-Nasab and Konstantaras [10] have formulated a problem using two isolated case of uniform and simple exponential distributions. The proposed model suggests a more realistic problem formulation by combining the two cases with truncated exponential distribution. It can be noted that neither uniform nor simple exponential distribution can truly reflect the case problem we are considering. However, by setting λ to zero in our problem, one can obtain the special case of simple exponential distribution in [10].

Lee [12] has considered a very different problem to determine the selling price, while the price in our model is an input data. Further, we have considered partial backorders (i.e. a combination of lost sales and backorders), but Lee has considered lots sales only. Moreover, we have considered a truncated exponential distribution for the time between two replenishments, but Lee's model has a completely different assumption on the replenishment.

As far as we know, there is no similar research in previous literature that considers truncated exponential distribution and special sale offer. We have also used the historical data of a pharmaceutical manufacturing company in Iran where the replenishment intervals can fit the statistical distributions such that: (1) all replenishment intervals are equal to or greater than t_{min}, (2) if all the replenishment intervals are decreased by t_{min}, then a simple exponential distribution is the best-fitted among other distributions by at least 95% confidence level. These observations show that replenishment intervals follow a truncated exponential distribution since this distribution starts from a value greater than zero such as $t_{min} = \lambda$ (see Fig. 2). We formulate the problem step by step and consider the following two cases:

Case *i*. Special order is placed.
Case *ii*. Special order is not placed.

Then, the net profit due to cost savings of placing a special order is maximized.

3. Problem formulation

In this section, we start by briefly reviewing the working conditions of the problem. Then, a list of input parameters and decision variables are introduced. A mathematical analysis of the problem under the following conditions is given:

- a single product type is purchased from a supplier,
- there is no limitation in the quantity of inventory stocked,
- demand rate is constant and deterministic over time,
- shortage with partial backorders is allowed (i.e. a portion of the shortages is lost and the others is backordered),
- the time between two replenishments obeys from a truncated exponential probability distribution,
- supplier offers special sale prices during some predefined time intervals such as Christmas,
- the system has the inventory ceiling of R, while in the special sale case, it is changed to R_s,
- there is a unique special sale opportunity,
- inventory does not deteriorate during the order cycle,
- other cost parameters are deterministic.

Next, the mathematical formulation is constructed in a step by step manner. Here are the modeling notations:

Input parameters	
D	demand rate of the product over time
T	time between two consecutive replenishments ($t_{min} \le T \le t_{max}$)
$f_T(t)$	probability distribution function (PDF) of T
$F_T(t)$	cumulative distribution function (CDF) of T
t_{min}	minimum expected value of the replenishment time (i.e. $t_{min} \le T$)
t_{max}	maximum expected value of the replenishment time (i.e. $T \le t_{max}$)
α	percent of the shortages that are backordered
t_d	the time it takes to finish the inventory after replenishment in the regular working cycles
$t_{d,s}$	the time it takes to finish the inventory after replenishment in the special sales
v	unit selling price of product
c	unit purchasing cost of product in the regular working cycles
c'	amount of decrease in unit purchasing cost in the special sales
c_s	unit purchasing cost of product in the special sales ($c_s = c - c'$)
h	unit inventory holding cost per unit time length
π	unit shortage cost for backordered items
π'	unit shortage cost for lost sales items
R	inventory ceiling in a regular working cycle
Decision variables	
B	maximum shortage in each working cycle
Q	order quantity in each regular inventory replenishment time
Q_s	order quantity in inventory replenishment time when dealing with special sales
EPN	average profit in a regular working cycle
EPS	average profit in the working cycle corresponding to special sales
G	gross savings from placing a special order
$\bar{I}$	average inventory level in a working cycle
$\bar{I}_s$	average inventory level during special sales
$\bar{B}$	average shortage level in a working cycle
$\bar{B}_s$	average shortage level during the special sales
$\bar{L}$	average lost sales in a working cycle
$\bar{L}_s$	average lost sales during the special sales
R_s	inventory ceiling in the special sales

The model is constructed based on two scenarios: (*i*) special order is placed, and (*ii*) special order is not placed. Fig. 1 illustrates the inventory variations over three working cycles. The problem considers an infinite planning horizon and an exponentially distributed replenishment interval. Our objective is to maximize profit during the special sale offer. Here, we calculate the terms *EPS* and