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Generating max-plus equations for efficient analysis of manufacturing flow lines

A. Seleim, H. ElMaraghy*

Intelligent Manufacturing Systems Centre, Univerity of Windsor, 401 Sunset Ave, Windsor, ON N9B 3P4, Canada

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ABSTRACT

An elegant mathematical tool, max-plus algebra, can be used to model manufacturing flow lines in the form of linear state-space-like equations for application in manufacturing systems analysis and control. A method for quick and efficient generation of the max-plus equations for manufacturing flow lines of any size or structure has been developed. The generated equations are used to model flow lines with finite buffers between stations as well as parallel identical stations.

A flow line is initially assumed to have infinite buffers for all stations and no parallel identical stations. An adjacency matrix is then used to encode the layout of the flow line and determine the relationships between different stations in the line. The max-plus equations for the line are then generated for the simplified line while capturing the line dynamics in two matrices that are function of the processing times of the different stations in the line. Additional terms are used to model finite buffers and the parallel identical stations.

The developed manufacturing systems modelling method using max-plus algebra is intuitive and easy to understand and code in software. It is a decision support tool which facilitates quick analysis of different configurations of manufacturing flow lines and assessment of several what-if scenarios. A case study is presented where the max-plus equations are generated for three possible assembly line configurations for a back flushing control valve. The effect of buffer sizes and changes in the assembly time of one of the stations on the total line idle time is analyzed using the generated model.

1. Introduction

Manufacturing systems fall under the category of Discrete Event Dynamic Systems (DEDS). For these systems, modelling tools include automata, petri-nets, Markov-chains, queuing networks, simulation, and max-plus algebra [1]. Among these models, max-plus algebra is the only tool that can model the system using linear algebraic equations analogous to conventional state-space linear equations [2]. Using these equations, real time control and parametric system analysis become possible.

The use of max-plus in modelling discrete event systems is fairly new starting in 1984 and since then it has been used in many manufacturing systems applications including: manufacturing systems modelling [3–5], performance evaluation [6–9], performance optimization [10], model predictive control [11–13] and just in time control [14].

Modeling simple manufacturing systems using max-plus equations is easy and intuitive; however, as the systems grow in size and/or have complicated structure, deriving the model equations becomes tedious, less intuitive and time consuming. In addition, deriving max-plus equations for manufacturing systems with finite buffers or parallel identical stations is not straight-forward or easy even for simple systems. The difficulty of deriving these equations limits the benefits of using max-plus algebra in modeling and controlling manufacturing systems especially when frequent changes in products or system configurations take place and the need for quickly assessing their effects and making decisions intensifies.

In this paper, a method for automatic generation of the max-plus system equations for flow lines is presented. The method can generate the equations for lines with complicated structures regardless of their size and can model finite buffers and parallel identical stations. Flow lines studied in this paper are assumed to have deterministic processing times and reliable stations. The first assumption is realistic for automated systems as well as semi-automated systems with palletized material handling where the process time variation is much less than the processing time and thus can be neglected. The second assumption is also realistic when

* Corresponding author. Tel.: +1 519 253 3000x3439.
E-mail addresses: hae@uwindsor.ca, imscadmin@uwindsor.ca (H. ElMaraghy).

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studying the normal short-term system operation with the objective of understanding and optimizing the system behavior rather than studying the long-term operation with the objective of planning system capacity where machines breakdown would have an effect. Preliminary results from this research have been recently presented in the 47th CIRP conference on manufacturing systems (CMS) in Windsor, Canada [15].

A review of related research is presented in Section 2. Section 3 presents a review the basics of max-plus algebra; Section 4 presents the method for generating the max-plus equations followed by a case study with an example of analysis and finally Section 6 presents the discussion and conclusions.

2. Related research

Several papers have been published focusing on facilitating the modeling of manufacturing systems using max-plus algebra. Doustmohammadi and Kamen [16] presented a procedure for direct generation of event-time max-plus equations for generalized flow shop manufacturing systems. The procedure is limited to flow shops with infinite buffers and cannot model parallel redundant machines. The procedure generates the equations directly only for serial flow lines with one station in each stage, otherwise the equations are generated for each machine separately, interconnection matrices which describe the flow of jobs through the line are derived and then the final equations are generated using matrix manipulations and recursions. Goto et al. [17] proposed a manufacturing systems representation that can account for finite buffers by adding relations between future starting times of jobs on a station and past starting times for the same and subsequent station. Imaev and Judd [18] used block diagrams which can be interconnected to form a manufacturing system model. This approach also assumes infinite buffer sizes and cannot model parallel redundant machines. Park and Morrison [9] presented a method for modelling flow lines with parallel redundant stations again by adding relations between future and past starting times on a station and the subsequent ones. However, their equations provide the processing starting time for jobs not stations, which is unusual in modeling manufacturing systems and causes the model variables and number of equations to grow with the number of jobs.

In summary, the literature is lacking a method for generating max-plus equations for complex flow lines which contain finite buffers and parallel identical stations.

3. Basics of max-plus algebra

Max-Plus algebra is an algebraic structure in which the two allowable operations are “maximization” and “addition”. In this section an introduction to the basic concepts and tools of the max-plus algebra will be presented.

Max-Plus algebra is defined over $\Re_{max} \to \{\Re \cup -\infty\}$ where $\Re$ is the set of real numbers. The two main algebraic operations are maximization, denoted by the symbol $\oplus$, and addition, denoted by the symbol $\otimes$ where:

$$a \oplus b = \max(a, b) \quad \forall a, b \in \Re_{max}$$

$$a \otimes b = a + b \quad \forall a, b \in \Re_{max}$$

The null element of the operation $\oplus$ is ε which is equal to $-\infty$, and the null element for the operation $\otimes$ is e which is equal to 0. This can by demonstrated by:

$$a \oplus \varepsilon = \max(a, -\infty) = a \quad \forall a \in \Re_{max}$$

$$a \otimes e = a + 0 = a \quad \forall a \in \Re_{max}$$

Similar to traditional algebra, both $\oplus$ and $\otimes$ are associative and commutative, and multiplication is left and right distributive over addition:

$$a \otimes (b \oplus c) = (a \otimes b) \oplus (a \otimes c) \quad \forall a, b, c \in \Re_{max}$$

$$(a \oplus b) \otimes c = (a \otimes c) \oplus (b \otimes c) \quad \forall a, b, c \in \Re_{max}$$

Max-plus algebra can be extended over matrices similar to conventional algebra. If $\boldsymbol{A}$ and $\boldsymbol{B}$ are two matrices with equal dimension then:

$$\boldsymbol{A} \oplus \boldsymbol{B} = \boldsymbol{C},$$

where $\boldsymbol{C}_{ij} = \boldsymbol{A}_{ij} \oplus \boldsymbol{B}_{ij}$. If the number of columns of $\boldsymbol{A}$ is equal to the number of rows of $\boldsymbol{B}$ equal to n, then:

$$\boldsymbol{A} \otimes \boldsymbol{B} = \boldsymbol{C},$$

where

$$\boldsymbol{C}_{ij} = \oplus_{k=1}^{n} (\boldsymbol{A}_{ik} \otimes \boldsymbol{B}_{kj}),$$

where $\oplus_{k=1}^{n} \boldsymbol{C}$ is maximization of all the elements of $\boldsymbol{C}$ over $k = 1$ to n.

Throughout the rest of the paper, the $\otimes$ operator will be omitted whenever its use is obvious, thus $a \otimes b \oplus c \otimes d$ will be written as $ab \oplus cd$.

According to Baccelli et al. [19], an equation in the general form:

$$\boldsymbol{X} = A\boldsymbol{X} \oplus B\boldsymbol{U} \quad (1)$$

where $\boldsymbol{X}$ is an $n \times 1$ vector of variables, $\boldsymbol{U}$ is an $m \times 1$vector of inputs, A is an $n \times n$ square matrix and B is an $n \times m$ matrix, has a solution:

$$\boldsymbol{X} = A^{*}B\boldsymbol{U} \quad (2)$$

where $A^{*}$is defined as:

$$A^{*} = \mathrm{e} \oplus A \oplus A^2 \oplus \ldots \oplus A^{\infty}.$$

A complete detailed description and analysis of the max-plus algebra can be found in [19,20].

4. Flow lines modeling

The presented method for modeling flow lines capitalizes on the observation that certain features of the line affect the final equations each in a specific way. For illustration, each specific feature will be presented separately to show its effect on the final equations and then the steps of arriving at the final equations for a general line will be presented followed by an example.

Modelling will start with a flow line with n serial stations, followed by n different lines merging (assembling) in one line, and then the effect of introducing parallel identical stations will be shown. Initially, infinite buffers are assumed before each station and then in Section 4.4 the effect of introducing finite buffers will be presented. Finally in Section 4.5 the whole model will be assembled and demonstrated by an example of a manufacturing flow line that contains serial and merging stations, parallel identical stations and finite buffers.

4.1. Modeling ‘n’ serial stations

The most common structure of a flow line is a serial structure with n processing stations, one input of incoming parts $\boldsymbol{U}$, and one output of finished products $\boldsymbol{Y}$ as shown in Fig. 1. Let $\boldsymbol{U_k}$, $\boldsymbol{Y_k}$, and $\boldsymbol{X_{i,k}}$ be the time at which the incoming parts are made available to the line, the time at which the finished product leaves the line and the starting time of processing on the ith station for the kth job respectively.