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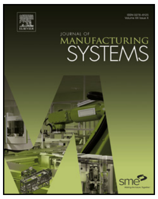

Technical Paper

An integrated inventory problem with transportation in a divergent supply chain under service level constraint

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ABSTRACT

This study investigates an integrated inventory problem with transportation in a single-vendor and multi-buyer divergent supply chain. The vendor manufactures a product and delivers the product to the buyers located in different locations by a fleet of vehicles of identical capacity. The external demands per unit time on the buyers are independent and normally distributed. The lead time components of the buyers, excluding transportation time, can be reduced at an added crash cost. A model has been formulated to minimize the total expected cost of the system associated with the production, inventory, transportation and lead time reduction to find the optimal production, inventory and routing decisions while satisfying the service level constraint of the buyers. We propose a coordinated two-phase iterative approach to solve the model, which has been illustrated through a numerical example.

1. Introduction

In today's competitive environments, costs and delays need to be constantly reduced to achieve optimal performance of a supply chain. As a result, the integrated decision of production, inventory and delivery operations has evolved as an effective strategy. Lei et al. [1] defined the problem of coordinating production, inventory, and delivery operations as the integrated production, inventory, and distribution routing problem. A review of the different kinds of coordination between production and distribution levels can be found in Ref. [2]. In general, optimally solving such an integrated problem is not easy due to its combinatorial nature, especially when vehicle routing is taken into account. To solve the problem of such kind, it is important to exploit the problem structure and use heuristics which can be coupled with an optimization approach that can generate good solutions quickly. Therefore, the problem of integration of production-inventory and distribution routing in a supply chain has attracted the attention of researchers (e.g. [3,4,5]) in recent years. The basic idea behind these models is to simultaneously optimize decision variables of different functions such as production, inventory and distribution routing that have traditionally been optimized sequentially requiring sufficiently large inventory buffer. However, this would lead to increased holding costs and longer lead times of products through supply chain. The pressure of reducing inventory and lead times in supply chain has forced companies to explore the integrated decisions of production, inventory and distribution.

Goyal [6] and Banerjee [7] initiated the idea of integration of production and inventory decisions in a single-vendor single-buyer supply chain and illustrated the benefit of integration. Subsequently, many researchers studied various vendor–buyer models under different assumptions. Some of the scholars also considered transportation explicitly in their models, such as Hoque and Goyal [8] developed an optimal solution procedure for a single-vendor single-buyer integrated inventory problem with unequal and equal-sized shipments from the vendor to the buyer under a capacity constraint of the transport equipment and fixed transportation cost for each shipment. Ertogral et al. [9] incorporated transportation cost into a single-vendor single-buyer supply chain considering all-unit-discount transportation cost structures with and without over declaration. Hsiao [10] considered both the transportation cost and transportation time for a two-stage supply chain consisting of a retailer and a supplier. Recently, Ben-Daya et al. [11] and Glock [12] provided an up-to-date review on integrated vendor–buyer models together with the mathematical description of the basic vendor–buyer models.

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In real-life, there are many examples in which a manufacturer (vendor) routes his/her product(s) to customers through several retailers (buyers). Joglekar and Tharthare [13] taking the lead proposed a multi-buyer model under finite production rate of the vendor. Later, Lu [14] studied the model assuming that each buyer orders a different item from the vendor with the objective of minimizing the total cost of the vendor subject to the maximum cost that the buyers may be prepared to incur. Yao and Chiou [15] proposed a heuristic for the single-vendor multi-buyer problem considering the same assumptions of Lu [14]. Abdul-Jalbar et al. [16] formulated the single-vendor multi-buyer problem in terms of the integer-ratio policies and developed a heuristic procedure to compute effectiveness of integer-ratio policies. Other authors (e.g. [17–20]) studied single-vendor multi-buyer integrated inventory models considering fixed cost of transporting a batch from the vendor to each buyer. Sarmah et al. [21] developed two models of coordination where multiple buyers receive supplies from a manufacturer. In the first model, the transportation cost is borne by the manufacturer, whereas in the second model, transportation cost is borne by the buyers. All these vendor–buyer models discuss about the integration of production and inventory decisions with fixed transportation cost ignoring the routing issue. In reality, such assumption is not valid and merely captures the transportation cost. As transportation cost constitutes a major part of the total operational cost, and so effect of transportation is required to be adequately reflected in final planning decisions, which is affected by vehicle routing and shipment lot size decisions.

The present study considers an analysis of integrated-inventory model with transportation cost in a single-vendor multi-buyer supply chain in an infinite planning horizon. The vendor produces a product in batch production environment at a finite production rate. The product is delivered to the buyers, facing stochastic demand, by a fleet of identical vehicles which combine the deliveries of several buyers into efficient routes and are dispatched simultaneously on all the routes at a common average ordering interval of the buyers. The quantity delivered per trip to the buyers is just enough to meet the average demand during their ordering intervals. The transportation cost structure of a vehicle consists of the operating cost which is proportional to the distance driven, plus a fixed cost which is incurred each time a tour of a vehicle is initiated.

When the demand is stochastic, lead time becomes an important issue and its control leads to many benefits. Shorter lead time reduce the safety stock requirements and the losses caused by stock-out, improves customer service level and increases the competitive advantage of business [22]. In fact, lead time usually consists of many components such as order preparation, order transit, setup time, waiting time, delivery time etc. [23]. In many practical situations, lead time can be shortened at the expense of extra cost which is known as lead time crashing cost. Liao and Shyu [24] first devised a probabilistic inventory model in which lead time was also considered as a decision variable. Subsequently, the concept of Liao and Shyu [24] for lead time reduction has been extended for the study of integrated inventory models in vendor–buyer supply chain under various settings by numerous scholars ([25–30,31,32,33]). In the present study, we also assume that the lead time of each buyer has several components in which all components are controllable and can be reduced at an added crashing cost, excluding the fixed transportation time component. The further assumptions which are made to describe the system and to formulate the model are listed in the next section.

The objective of the proposed model is to minimize the joint total expected cost of ordering, production setup, inventory holding, lead time crashing and vehicle routing. Instead of considering stock-out cost terms in the joint total expected cost expression, service level constraint (SLC) in terms of fill rates corresponding to each buyer are included. The optimal order quantity (delivery size), lead time, and safety factor of buyers and shipment frequency per production cycle of vendor and efficient routes are determined simultaneously by solving the combined integrated inventory problem and vehicle routing problem (VRP) using a coordinated two-phase iterative approach.

The outline of the remainder of the paper is as follows. The next section describes the notations and assumptions used throughout this paper. In Section 3, we provide an introduction to the problem and formulate the mathematical model. A solution technique is developed to obtain the optimal solution of the proposed model in Section 4. A numerical example is presented in Section 5 to illustrate the proposed solution procedure. Finally, conclusions are discussed in Section 6.

2. Notations and assumptions

The following notations and assumptions are used to define the problem. Some additional notations and assumptions will be listed later when they are needed.

2.1. Notations

N number of buyers indexed from 1 to N; index 0 denotes the vendor
$E()$ mathematical expectation
x^+ maximum value of x and 0, i.e. $x^+ = \max\{x, 0\}$.
For the ith buyer ($i = 1, 2, \ldots, N$)
D_i average demand per unit time
A_i ordering cost per order
C_{bi} unit purchase cost
h_i holding cost rate (per monetary unit invested in inventory) per unit time
r_i reorder point
Q_i order quantity (decision variable)
k_i safety factor (decision variable)
L_i length of lead time (decision variable)
T_i transportation time for an order to arrive at buyer i from the vendor (decision variable)
α_i proportion of demands that are not met from stock so $(1-\alpha_i)$ is the service level
$B_i(r_i)$ expected demand shortages at the end of buyer's cycle
X_i lead time demand, which is normally distributed with finite mean D_iL_i and standard deviation $\sigma_i\sqrt{L_i}$, where σ_i denotes the standard deviation of demand per unit time, $X_i \sim N(D_iL_i, \sigma_i\sqrt{L_i})$.
For the vendor