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Technical Paper

Waste minimization in irregular stock cutting

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ABSTRACT

This paper addresses a category of two dimensional NP-hard knapsack problem in which a given convex/non-convex planner items (polygons) have to be cut out of a single convex/non-convex master surface (stock). This cutting process is found in many industrial applications such as sheet metal processes, home-textile, garment, wood, leather and paper industries. An approach is proposed to solve this problem, which depends on the concept of the difference between the area of a collection of polygons and the area of their convex hull. The polygon assignment inside the stock is subjected to feasibility tests to avoid overlapping, namely, angle test, bound test, point inclusion and polygon intersection test. An iterative scheme is used to generate different polygon placements while optimizing the objective function. Computer software is developed to solve and optimize the problem under consideration. Few examples are conducted for different combinations of convex, non-convex items and stocks. Well-known benchmark problems from the literature are tested and compared with our approach. The results of our algorithm have an interesting computational time and can compete with the results of previous work in some particular problems. The computational performance of the developed software indicates the efficiency of the algorithm for solving 2-D irregular cutting of non-convex polygons out of non-convex stock.

1. Introduction

The process of packing a given set of 2-dimensional planner items into a bin of minimal area is an intricate nesting problem that has been known in many applications. These applications arise in many industries such as clothing and textile, steel construction, wood, glass, leather and paper industries. In fact, a great deal of literature is found on such a problem [1–3].

Nesting problems are considered NP-hard due to their difficulty where no exact methods have been reported in the literature, instead, only heuristic procedures have been developed [4,5,22,28,29]. The heuristic methods presented in the literature deal mostly with class of problems in which the objective is to minimize the length of the master surface used. Such approaches do not always suite problems where limited master surface is used such as hides or sheet metals. Packing usually suites the division of problems in which stock material comes in rolls. However, for the case when limited length or bounded stocks are used, bin packing will result in degradation in the utilization of material. Instead, the idea of knapsack problem can be put into practice to serve such purpose. The interested reader can come across appealing and precise surveys about nesting [6,12].

Bin packing problem can be seen as a special case of the cutting stock problem. In fact, when the number of bins is restricted to one and each item is characterized by both area and value, the problem of maximizing the value of items that can fit in the bin is known as the knapsack problem which characterizes our work in this paper.

Generally in most nesting problems, two directions are followed; the first deals with nesting strategies and their evaluation criteria, such as minimizing area, minimizing length, minimizing overlap and the evaluation criteria, i.e., waste, overlap, and distance [8,9]. The second research direction addresses the search methods, that is, the order of item packing. For instance, Gomes and Oliveira [10] presented a search technique based on a 2-exchange neighborhood generation mechanism, simulated annealing [11], hill climbing [9], nesting with defects [5], etc.

Quite a few studies were presented to address the problem of nesting irregular shapes [7,13,14]. Most of this research implemented the idea of No Fit Polygon (NFP) to detect potential overlap between polygons [15]. As a matter of fact, the computational time required to detect overlap has been significantly reduced with the implementation of NFP algorithms which contribute most to the packing computational time. The introduction of NFPs has motivated most of the recent work in nesting problems.

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Remarkable breakthroughs in algorithms for nesting problems have been achieved in recent years. Compared to previous studies, these solutions have noticeably enhanced the effect and speed of nesting results. Remarkable methods were presented by Crispin et al. [16], Burke et al. [9], Gomes and Oliveira [11], Egeblad et al. [17], Lee et al. [18], Imamichi et al. [19], Sato et al. [20] and Alves et al. [21]. In particular, Crispin et al. [16] described two genetic algorithms to solve the LNP arising in the shoe manufacturing industry based on different coding methodologies. The problem tackled by Crispin took directionality constraints into account where imposing specific directions for the pieces in given regions of the hides reduces the solution space.

The heuristic developed by Egeblad et al. [17] is based on a simple local search plan in which the neighborhood is simply any vertical or horizontal translation of a given polygon from its present location. A guided local search is applied to escape local minima. Lee et al. [18] describes a heuristic for placing irregular shapes on multiple sheets. They use a simple nesting rule to place the shapes one after another. Such placement is improved later by movement and rotation operations. In [18], only one single experiment with one instance is presented for nesting irregular shapes. Based on nonlinear programming model, Imamichi et al. [19] combined iterative local search with a swap procedure and a separation algorithm to solve the overlapping minimization problem. Meanwhile, some modern studies adopted the collision free region to determine feasible layouts in containers with determined dimensions [20].

Alves et al. [21] explored different strategies for grouping, selecting and placing the pieces inside the hides of leather, later a quality evaluation of the placement is carried out. Alves approach depends on the computation of the no-fit polygons. The quality of placement was measured using a linear combination of the values of some attributes, particularly; the area, degree of irregularity, degree of concavity, the ratio between length and width and quality zones. Specific weights have to be allocation for each quality attribute which makes it vulnerable to human interaction.

In our study, the goal is to address cutting convex and non-convex items out of a convex or non-convex stock. The proposed methodology suites real industrial applications in the sense that it imitates different blank/stock shapes.

The paper is organized as follows: the methodology is presented in Section 2 in which we describe the problem, the waste minimization, polygons representation, the algorithm steps, the objective function and the geometric transformation. The implementation of the algorithm is illustrated in Section 3 along with the pseudo code and the different feasibility checks. Section 4 demonstrates the built computation library and the CAD package. Next in Section 5, the model parameters are discussed followed by the experimental results of different knapsack problems in Section 6. Comparisons with literature benchmark problems are provided in Section 7 followed by the conclusions in Section 8.

2. Methodology

2.1. Minimizing waste

As in any classical 2-dimenstional knapsack problem, our objective is to maximize the aggregate value of the included items (polygons) within a stock such that the items are bounded by the stock with no overlap. Here, the value to be maximized is the sum of the items' areas. We will simply attempt to cut as many types of planner items as possible out of a convex/non-convex stock.

The idea of minimizing the waste in our approach depends on the concept of the difference between the area of a collection of polygons and the area of their convex hull. Convex hull is the outer band that includes a set of points, or by definition, the convex hull or convex envelope of a set F of points (vertices) in the Euclidean space is the smallest convex set that contains F. Here, F includes all the vertices of a set of non-overlapping polygons. An example of convex hull is illustrated in Fig. 1. The polygon "A" has an area of 3 while its convex hull has an area of 3.5 squared units. Note that convex polygons will always have the same area as their convex hull, such as rectangles, pentagons, hexagons, etc. Hence, these polygons will have a zero difference between their areas and the areas of their convex hull.

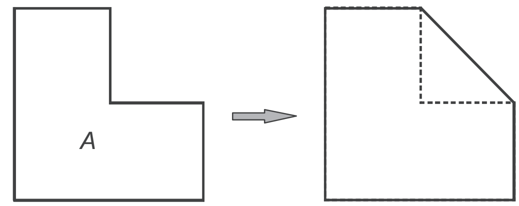

Fig. 1. A non-convex polygon and its convex hull.

Consider a simple polygon placement procedure in which a unit-square polygon "B" is to be placed on all the vertices of polygon "A" that is, each vertex of "B" will be positioned on each vertex of A while avoiding overlap, Fig. 2. Here, whenever a vertex of A is positioned on vertices of B, a new configuration will result. Obviously, some of these configurations may be infeasible due to the polygons overlap or due to exceeding the stock boundaries. From a better compaction point of view, the configuration with the minimum convex hull area can be considered as one of the finest orientations. Fig. 2 shows some of the orientations and their related convex hulls. Clearly, when the squared box "B" is positioned at the cut-off in polygon "A" (the most right) it will result in the minimum convex hull area, which equals to 4 squared units in this example.

Another example is given in Fig. 3. Common sense reveals that, among all the different feasible orientations, positioning the triangle as shown in configuration 2 is more preferred as it reduces the bounding area. The convex hull in configuration 2 is less in area as compared to configuration 1. Therefore, as a part of our proposed methodology, the algorithm objective function will include the minimization of the convex hull area of any collection of polygons.

On the other hand, reducing the convex hull area does not necessarily mean maximizing the utilization of stock area. For instance, consider the situation when there are three polygons as presented in Fig. 4. Placing polygon "C" in the large slot is more appropriate than placing polygon "B" as the total area used inside the convex hull will be larger. This condition is achieved despite that the convex hull for both configurations is the same.

Accordingly, it is more convenient to use the difference between the area of the convex hull containing all the polygons and the area of the polygons. Nonetheless, in some situations different orientations may result in the same difference between the convex hull and polygons' areas, although there might be some preference for one over another. Fig. 5 demonstrates the idea that the three polygons (A–C) have to be attached to each other in such a way that will help minimizing the gaps. For simplicity it is assumed that each polygon is constructed out of squared units. Positioning either polygon "B" or "C" inside the cut-off of polygon "A" will result in the same difference between convex hulls and polygon areas. Hence, the algorithm objective function is modified to consider the relative difference in the areas as compared to the area of the convex hull. In this example, the difference is one unit in the configurations 1 and 2. However, it could be said that configuration 2 should be more preferred than configuration 1. In order to impose a preference to configuration 2, the relative difference is used. Here, the relative