Available online at www.sciencedirect.com

ScienceDirect

Procedia CIRP 50 (2016) 228 – 233

www.elsevier.com/locate/procedia

26th CIRP Design Conference

Mathematical Underpinnings for Achieving Design Functional Requirements

Hilario (Larry) Oh

MIT Park Center for Complex Systems, Cambridge, MA, USA

* Corresponding author. Tel.: +1-248-705-0003; E-mail address: ohlarry@yahoo.com

Abstract

The two types of failure to achieve design functional requirements (FRs) are: Type I, the design cannot hit the FR targets; Type II, it cannot hit them consistently. The causes are due to inter-dependence among the FRs in Type I; and due to build and usage variability of the design in Type II. In this paper, we develop a mathematical understanding for the two types of failures. The underpinnings are Jacobian matrix of FR with respect to input variables for Type I failure; and Jacobian matrix of FR with respect to noise (sources of variability) variables for Type II. Since Independence axiom and Information axiom of Axiomatic Design relate to the interdependence and variability of FRs, the understandings developed herein also serve as the mathematical underpinnings for the two design axioms. The design of snap-fit is used to illustrate the concept and process involved.

© 2016 The Authors. Published by Elsevier B.V. This is an open access article under the CC BY-NC-ND license (http://creativecommons.org/licenses/by-nc-nd/4.0/).
Peer-review under responsibility of the organizing committee of the 26th CIRP Design Conference

Keywords: Functional dependence, Jacobian determinant, Axiomatic Design, Functional requirement, Design parameter.

1. Introduction

Prior to the late 1970's, design was looked upon more as an art than a science. There was no scientific basis to assess goodness of a design, so that we had to resort to build and test for assessment. In 1979, Axiomatic Design Theory (AD) was introduced to make design more of a science than an art. It introduced axioms to define goodness of design and guided the designers through the design process. The axioms are assumed to be self-evident truths for which there are no counter-examples or exceptions. They cannot be proven nor derived from other laws or principles of nature, [1].

AD is a big step toward the goal of establishing design as a science. In spite of its successful applications, rejection of AD persists in part of design community. The primary reason is the axiomatic assumption it imposes. Typical criticisms are: "AD people invoke axioms to avoid proof of theory"; "AD is not a mathematically valid method". We understand and feel for these criticisms. Thus in this paper, we develop a mathematical basis in place of axiomatic assumption to further advance design as a science.

First, we establish the mathematical basis by considering the primary objective in design. It is to achieve the target values of the design FRs with reduced variation around them. Failure to achieve the objective can occur in two ways:

Type I – functional coupling makes adjustment of design parameters (DPs) to achieve target values of FRs difficult; and

Type II – corruption by 'noise' that causes variation in FRs.

In Section 2, we cast the adjustment of DPs to achieve targets of FRs as a root-finding problem. As a result, the crux of Type I failure is revealed in Section 3 as the failure to find roots due to functional dependence among the FRs. In Section 4, we introduce the concept of noise to express Type II failure in terms of bias and spread of FRs induced by noise. This enables us to treat Type II failure as an optimization problem minimizing spread subject to the constraint that bias equals zero. In Section 5, we discuss the relevance of above findings to Axiomatic Design. In Section 6, the design of snap-fit is used to demonstrate the mathematical understandings and their implementation. Some of the concepts herein were developed earlier in [2]. We end with concluding remarks in Section 7.

2. Achieving the targets of FRs is root-finding

Usually, a design has multiple functional requirements FRs. These FRs are realized with physical entities which we label as design parameters DPs through physical laws that relate FRs to DPs which we denote as $f_k(\cdot)$, k = 1, 2, ⋯, n.

$$\begin{aligned} FR_1 &= f_1\left(DP_1,\cdots,DP_m\right) \\ &\vdots \\ FR_n &= f_n\left(DP_1,\cdots,DP_m\right) \end{aligned}$$

Or in vector form,

$$\mathbf{FR} = \mathbf{f}\,(\mathbf{DP})$$

2212-8271 © 2016 The Authors. Published by Elsevier B.V. This is an open access article under the CC BY-NC-ND license (http://creativecommons.org/licenses/by-nc-nd/4.0/).
Peer-review under responsibility of the organizing committee of the 26th CIRP Design Conference
doi:10.1016/j.procir.2016.04.141

In the above and hereafter, bolded quantities denote vectors, bracketed quantities denote matrices and **f** (•) denotes vector valued functions.

Note that the target values of functional requirements $\mathbf{FR}^*$ are known. Thus, the task of adjusting **DP** to achieve target values $\mathbf{FR}^*$ is equivalent to finding the root $\mathbf{DP}^*$ that satisfies:

$$\mathbf{DP}^* = \mathbf{f}^{-1}(\mathbf{FR}^*)$$

Frequently, the vector valued function **f** (**DP**) is nonlinear. So that it is not possible to solve $\mathbf{f}^{-1}(\mathbf{FR}^*)$ analytically. Instead numerical methods are used to approximate the solution. One such method is the Newton-Raphson in which the nonlinear problem is replaced by a succession of linear problems whose solutions converge to the solution of the non-linear problem. Specifically, to find **DP** that satisfies

$$\mathbf{f}(\mathbf{DP}) - \mathbf{FR}^* = \mathbf{0}, \qquad (1)$$

we approximate the function **f**(**DP**) by its first-order Taylor expansion about $\mathbf{DP}^k$ to obtain:

$$\mathbf{f}(\mathbf{DP}) \approx \mathbf{f}(\mathbf{DP}^k) + [\mathrm{J}^{\mathrm{DP}}](\mathbf{DP} - \mathbf{DP}^k) \qquad (2)$$

In the above, the superscript k denotes the kth iteration and $[\mathrm{J}^{\mathrm{DP}}]$ is the n x n Jacobian of **f**(**DP**) with respect to **DP** evaluated at $\mathbf{DP}^k$ shown below.

$$\left[\mathrm{J}^{\mathrm{DP}}\right] = \begin{bmatrix} \dfrac{\partial \mathrm{FR}_1}{\partial \mathrm{DP}_1} & \cdots & \dfrac{\partial \mathrm{FR}_1}{\partial \mathrm{DP}_n} \\ \vdots & \dfrac{\partial \mathrm{FR}_i}{\partial \mathrm{DP}_j} & \vdots \\ \dfrac{\partial \mathrm{FR}_n}{\partial \mathrm{DP}_1} & \cdots & \dfrac{\partial \mathrm{FR}_n}{\partial \mathrm{DP}_n} \end{bmatrix}_{\mathbf{DP}^k} \qquad (3)$$

Note that the dimension of **FR** n is generally not equal to the dimension of **DP** m. If $n > m$, Type I failure will occur since there is insufficient **DP** to satisfy **FR**. If $n \le m$, then we choose one of the $C(m, n)$ combinations of Jacobian $[\mathrm{J}^{\mathrm{DP}}]$ with dimension n x n that ensures functional independency as described in the next section.

Solving for the root of Equation (1), we have

$$\mathbf{f}(\mathbf{DP}^k) + [\mathrm{J}^{\mathrm{DP}}]\,(\mathbf{DP} - \mathbf{DP}^k) - \mathbf{FR}^* = \mathbf{0}$$

$$\mathbf{DP}^{k+1} = \mathbf{DP}^k - \left[\mathrm{J}^{\mathrm{DP}}\right]^{-1}\left[\mathbf{f}(\mathbf{DP}^k) - \mathbf{FR}^*\right] \qquad (4)$$

Iteration is made of Equation (4) with k = 0, 1, 2 … until $\left\|\mathbf{DP}^{k+1} - \mathbf{DP}^k\right\|$ is less than a desired accuracy. At which point $\mathbf{DP}^{k+1}$ serves as the root $\mathbf{DP}^*$ to Equation (1). It is the value with which to tune **FR** to its target value $\mathbf{FR}^*$.

3. Linking root-finding to functional dependency

The crux to Type I failure is revealed in Equation (4). If the determinant $|\mathrm{J}^{\mathrm{DP}}| = 0$, then the inverse of the Jacobian $[\mathrm{J}^{\mathrm{DP}}]^{-1}$ does not exist. So that no root can be found that will satisfy Equation (1). Namely, **FR** cannot achieve its target value. The condition that leads to $|\mathrm{J}^{\mathrm{DP}}| = 0$ can be traced to the functional dependence among the FRs. In the next section, we derive the mathematics surrounding this condition. The derivation is confined to two FRs involving two DPs. Still, the logic holds true for n FRs involving n DPs, $n > 2$.

Consider a design with two FRs involving two DPs. They are related via physical laws $\mathrm{f}_1(\mathrm{DP}_1, \mathrm{DP}_2)$ and $\mathrm{f}_2(\mathrm{DP}_1, \mathrm{DP}_2)$:

$$\mathrm{FR}_1 = \mathrm{f}_1(\mathrm{DP}_1, \mathrm{DP}_2)$$
$$\mathrm{FR}_2 = \mathrm{f}_2(\mathrm{DP}_1, \mathrm{DP}_2)$$

A Taylor series expansion of FR_1 and FR_2 about their targets FR_1^* and FR_2^* retaining only the first-order terms gives,

$$\mathrm{FR}_1 \approx \mathrm{FR}_1^* + \frac{\partial \mathrm{FR}_1}{\partial \mathrm{DP}_1}\Delta \mathrm{DP}_1 + \frac{\partial \mathrm{FR}_1}{\partial \mathrm{DP}_2}\Delta \mathrm{DP}_2 \qquad (5)$$

$$\mathrm{FR}_2 \approx \mathrm{FR}_2^* + \frac{\partial \mathrm{FR}_2}{\partial \mathrm{DP}_1}\Delta \mathrm{DP}_1 + \frac{\partial \mathrm{FR}_2}{\partial \mathrm{DP}_2}\Delta \mathrm{DP}_2 \qquad (6)$$

We eliminate $\Delta \mathrm{DP}_2$ by subtracting $\partial \mathrm{FR}_1/\partial \mathrm{DP}_2$ x Equation (6) from $\partial \mathrm{FR}_2/\partial \mathrm{DP}_2$ x Equation (5):

$$\begin{aligned}\left(\mathrm{FR}_1 - \mathrm{FR}_1^*\right)\frac{\partial \mathrm{FR}_2}{\partial \mathrm{DP}_2} - \left(\mathrm{FR}_2 - \mathrm{FR}_2^*\right)\frac{\partial \mathrm{FR}_1}{\partial \mathrm{DP}_2} \\ = \left(\frac{\partial \mathrm{FR}_2}{\partial \mathrm{DP}_2}\frac{\partial \mathrm{FR}_1}{\partial \mathrm{DP}_1} - \frac{\partial \mathrm{FR}_1}{\partial \mathrm{DP}_2}\frac{\partial \mathrm{FR}_2}{\partial \mathrm{DP}_1}\right)\Delta \mathrm{DP}_1\end{aligned} \qquad (7)$$

Note that $\left(\frac{\partial \mathrm{FR}_2}{\partial \mathrm{DP}_2}\frac{\partial \mathrm{FR}_1}{\partial \mathrm{DP}_1} - \frac{\partial \mathrm{FR}_1}{\partial \mathrm{DP}_2}\frac{\partial \mathrm{FR}_2}{\partial \mathrm{DP}_1}\right)$ is determinant $|\mathrm{J}^{\mathrm{DP}}|$:

$$\left|\mathrm{J}^{\mathrm{DP}}\right| \equiv \begin{vmatrix} \dfrac{\partial \mathrm{FR}_1}{\partial \mathrm{DP}_1} & \dfrac{\partial \mathrm{FR}_1}{\partial \mathrm{DP}_2} \\ \dfrac{\partial \mathrm{FR}_2}{\partial \mathrm{DP}_1} & \dfrac{\partial \mathrm{FR}_2}{\partial \mathrm{DP}_2} \end{vmatrix}$$

If $|\mathrm{J}^{\mathrm{DP}}| = 0$, then per Equation (7), FR_2 is a function of FR_1 :

$$\left(\mathrm{FR}_1 - \mathrm{FR}_1^*\right)\frac{\partial \mathrm{FR}_2}{\partial \mathrm{DP}_2} - \left(\mathrm{FR}_2 - \mathrm{FR}_2^*\right)\frac{\partial \mathrm{FR}_1}{\partial \mathrm{DP}_2} = 0.$$

$$\mathrm{FR}_2 = \mathrm{FR}_2^* + \left(\mathrm{FR}_1 - \mathrm{FR}_1^*\right)\left(\frac{\partial \mathrm{FR}_2}{\partial \mathrm{DP}_2}\right)\left(\frac{\partial \mathrm{FR}_1}{\partial \mathrm{DP}_2}\right)^{-1} = \mathrm{FR}_2\left(\mathrm{FR}_1\right)$$

Hence,

$|\mathrm{J}^{\mathrm{DP}}| = 0$ implies functional dependence of FR_2 on FR_1. (I)

We next prove the converse is true. We start with the formal definition of functional dependency. Namely, FR_2 is functionally dependent on FR_1 if it is a function of FR_1 :

$$\mathrm{FR}_2 = \mathrm{FR}_2(\mathrm{FR}_1)$$

Download English Version:

https://daneshyari.com/en/article/1698154

Download Persian Version:

https://daneshyari.com/article/1698154

Daneshyari.com