Available online at www.sciencedirect.com
ScienceDirect
Procedia CIRP 46 (2016) 242 – 245
www.elsevier.com/locate/procedia

77th HPC 2016 – CIRP Conference on High Performance Cutting

Modeling and Measurement of Micro End Mill Dynamics Using Inverse Stability Approach

E.E. Yılmaz[a], E. Budak[b], H.N. Özgüven[a]

[a]*Department of Mechanical Engineering, Middle East Technical University, 06531 Ankara, Turkey*
[b]*Manufacturing Research Laboratory, Sabancı University, 81474 İstanbul, Turkey*

* Corresponding author. Tel.: +90-533-348-2686 . *E-mail address:* ersoyylmaz@gmail.com

Abstract

Micro-milling applications require high precision and dimensional accuracy. Chatter vibrations arising from unstable cutting conditions cause poor surface finish and damage to the cutting tools. Tool point frequency response functions (FRF); needed to generate stability diagrams, cannot be determined experimentally due to very small tool size. In this study an analytical model for tool point FRF of micro end mills is presented. An inverse algorithm is proposed to correct geometric representation and damping of the tool. The model and the correction methodology presented improve productivity and part quality in micro-milling through accurate prediction of chatter stability limits, enabling better cutting parameter selection.

Keywords: Micro-milling; Chatter Stability; Machine Tool Dynamics

1. Introduction

Chatter is a common problem in machining operations causing poor surface finish and damage to cutting tool. In order to avoid chatter stability diagrams are needed. One of the requirements for generating the stability diagrams is tool point frequency response function (FRF). Experimental modal analysis is the most common method of determining the tool point FRF [1]. However, due to their fragile structures, impact testing is not suitable for micro-milling tools. Therefore other methods must be utilized in order to obtain tool point FRFs of micro-milling tools. Schmitz et al. [2-3] proposed the receptance coupling method to obtain tool point FRFs of multi segmented Euler-Bernoulli beams. Ertürk et al. [4] modeled the segments of the structure with Timoshenko beam model. In micro-machining, Mascardelli et al. [5] used the receptance coupling method to couple the micro tool FRF obtained by finite element method with the spindle-holder FRF measured with modal experiments. Rahnama et al. [6] worked on the cutting process dynamics and considered process damping effects in stability diagram generation. Park et al. [7] modeled the uncertainty during cutting process by considering cutting parameters as variables that are changing within certain range. Jin et al [8] measured the FRF at fragile tool tip with specially designed piezo-actuator which is capable of exciting the tool tip without damaging it. Bediz et al [9] also presented an experimental approach to measure the tool tip FRF with a custom made excitation system.

In this paper, a new method to obtain the tool point FRFs of micro tools is presented. The presented inverse stability analysis, which enables us to obtain the modal parameters from the chatter tests, is verified with experiments. The analytical model for obtaining tool point FRF presented by Özşahin et al. [10] is updated according to the results of the inverse stability analysis. A considerable increase in the damping ratio of the cutting tool is observed.

2. Inverse Stability Method for Micro Tools

As proposed by Budak et al. [11] and Özşahin et al. [12], the

doi:10.1016/j.procir.2016.04.114

axial depth of cut and the chatter frequency can be calculated as follows;

$$a_{\lim} = -\frac{2\pi\Lambda_R}{NK_t}(1+\kappa^2) \quad (1)$$

$$\omega_c T = \cos^{-1}\left(\frac{\kappa^2-1}{\kappa^2+1}\right) \quad (2)$$

where,

$$\Lambda = (\Lambda_R + \Lambda_I i) = -\frac{1}{2a_0}\left(a_1 \pm \sqrt{a_1{}^2 - 4a_0}\right) \quad (3)$$

$$a_0 = G_{xx}G_{yy}\left(a_{xx}a_{yy} - a_{xy}a_{yx}\right) \quad (4)$$

$$a_1 = a_{xx}G_{xx} + a_{yy}G_{yy} \quad (5)$$

$$a_{xx} = \frac{1}{2}\left[\cos 2\phi - 2r\phi + r\sin 2\phi\right]_{\phi_{st}}^{\phi_{ex}} \quad (6)$$

$$a_{xy} = \frac{1}{2}\left[-\sin 2\phi - \phi + r\cos 2\phi\right]_{\phi_{st}}^{\phi_{ex}} \quad (7)$$

$$a_{yx} = \frac{1}{2}\left[-\sin 2\phi + 2\phi + r\cos 2\phi\right]_{\phi_{st}}^{\phi_{ex}} \quad (8)$$

$$a_{yy} = \frac{1}{2}\left[-\cos 2\phi - 2r\phi - r\sin 2\phi\right]_{\phi_{st}}^{\phi_{ex}} \quad (9)$$

$$\kappa = \frac{\Lambda_I}{\Lambda_R} \quad (10)$$

Here, *Gxx* and *Gyy* are the tool point FRFs in x and y directions, respectively. Chatter frequency is represented by ω_c, and the axial depth of cut is represented by a_{lim}. N is the number of cutting teeth, K_t is the tangential cutting force coefficient, r is the ratio of the radial and tangential cutting force coefficients, ϕ_{st} and ϕ_{ex} are the start and exit angles of the cutting tooth, T is the tooth period. The tool point FRF in each direction can be expressed as follows;

$$G_{xx} = \sum_{j=1}^{n}\frac{A_j}{\omega_j{}^2-\omega^2+i2\xi_j\omega\omega_j} \quad (11)$$

$$G_{yy} = \sum_{j=1}^{n}\frac{A_j}{\omega_j{}^2-\omega^2+i2\xi_j\omega\omega_j} \quad (12)$$

In the inverse stability approach the modal parameters in the FRF are the unknowns and the chatter frequency and depth of cut are the values obtained experimentally. The modal parameters can be identified by equating the experimentally obtained chatter frequency and corresponding axial depth of cut with their analytical definitions.

$$a_{lim}^{analytical} = a_{lim}^{experimental} \quad (13)$$

$$\omega_c^{analytical} = \omega_c^{experimental} \quad (14)$$

For each mode in the two orthogonal planes there exists 3 unknowns which are modal constant, natural frequency, and damping ratio. However, equating experimentally and analytically obtained chatter frequencies and axial depths of cuts, yields only 2 equations. On the other hand, it is possible to reduce the number of unknowns by considering the characteristics of stability diagrams. It is known that usually a dominant mode in the FRF is responsible for the stability diagram. Thus, it is possible to assume one mode in the FRF and remove the summation sign in equations (11) & (12). Doing so would reduce the total number of unknowns to 6, which are the 3 modal parameters for x and y directions each.

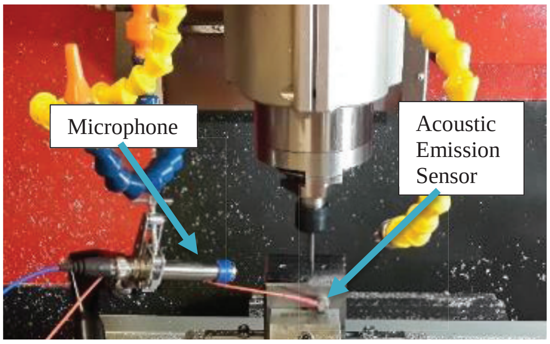

Fig. 1. Experimental Setup

The number of unknowns can be further reduced by recognizing that in micro-milling tool point FRFs in x and y directions are the same. Thus the equations (11) & (12) reduce to a single equation after replacing the frequency ω in the FRF expressions with chatter frequency ω_c.

$$G = \frac{A}{\omega_n{}^2-\omega_c{}^2+i2\xi\omega\omega_n} \quad (15)$$

The only unknowns in equation (15) are the 3 modal parameters. Therefore, 2 equations with 3 unknowns can be obtained. In order to solve this problem the number of equations will be increased by performing more experiments. This is possible since doing experiments at different spindle speeds and depth of cuts would give us a new set of experimental chatter frequency and depth of cut in equations (13) and (14).

By rearranging the equations (1) and (10), it is possible to express Λ_R *and* Λ_I as follows.

$$\Lambda_R = -\frac{a_{lim}NK_t}{2\pi(1+\kappa^2)} \quad (16)$$

$$\Lambda_I = \kappa\Lambda_R \quad (17)$$

Substituting equations (4), (5), (16) and (17) into equation (3) and replacing FRF terms G_{xx} and G_{yy} with equation (15) would give us following complex equation.

$$\frac{\alpha a_{lim}NK_t}{\pi(1+\kappa^2)}(1+i\kappa) = \frac{\beta\left(\frac{A}{\omega_n{}^2-\omega_c{}^2+i2\xi\omega\omega_n}\right) \pm \sqrt{(\beta^2-4\alpha)\left(\frac{A}{\omega_n{}^2-\omega_c{}^2+i2\xi\omega\omega_n}\right)^2}}{\left(\frac{A}{\omega_n{}^2-\omega_c{}^2+i2\xi\omega\omega_n}\right)^2} \quad (18)$$

where,

$$\alpha = a_{xx}a_{yy} - a_{xy}a_{yx} \quad (19)$$

$$\beta = a_{xx} + a_{yy} \quad (20)$$

Equation (18) is a complex equation with 3 unknowns which are ω_n (natural frequency), ξ (damping ratio), and A (modal