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An Interpolation Concept for Linear Blending based on Cornu Spiral

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Abstract

Feedrate reduction on path transitions with different tangents can be avoided by fitting blending curves. In this paper, a concept for applying Cornu spirals as blending curve for linear path transition is introduced. As opposed to interpolation methods in current CNC systems, the curvature profile of Cornu spiral is linear. In machining context, this can ensure a jerk limited machining while maintaining high feedrate. In an experimental setting, Cornu spiral is compared with polynomial spline to outline the benefit of applying Cornu spiral as blending curve.

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1. Introduction

Bézier curve, polynomial, B-spline and NURBS are the de facto standard interpolation methods used in CNC systems for fitting blending curves. By definition, blending, also known as continuous mode [1], is a method for smoothing the path and feedrate profile on sharp path transitions with different tangent vectors. By fitting a blending curve, the feedrate of the current NC block can be maintained for the successive block at the expense of machining accuracy (Fig. 1). The aforementioned interpolation methods can ensure the smoothness of the path, i.e. tangent and curvature continuity. In context of machining task, this results in a smooth velocity profile and a centripetal jerk-free motion. An additional requirement for the blending process is to maintain a constant material removal rate (MRR) to ensure the surface quality. Mathematically, MRR is described as a function of the feedrate v, depth of cut δ, curvature κ, and tool radius d, $MRR = v\delta\,[1 + \kappa(d - \delta/2)]$ [3]. A constant MRR is commonly achieved by varying the feedrate depending on the current curvature value [3]. The curvature profile of the aforementioned interpolation methods is, however, not controllable or, in other words, not linear over the arc length, so that constant MRR is difficult to maintain. In this paper, the possibility of applying Cornu spiral as blending curves between linear segments is examined.

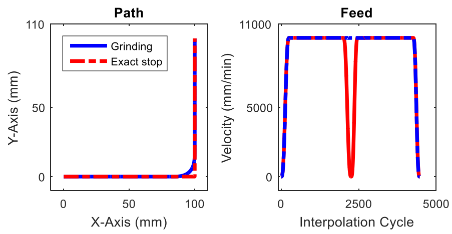

Fig. 1. smoothing of path and feedrate by fitting a blending curve.

2. Cornu spiral

Cornu spiral, also known as Clothoid or Euler spiral, is a planar curve defined in parameter form. Particular properties of Cornu spirals are the linear increase of the curvature over the arc length and the exact arc length parametrization. Cornu spiral can be parametrized differently, either by its tangent θ,

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curvature κ, arc length s, or the so called Cornu parameter t. For blending curves with tangent and curvature continuity requirement, the ideal parametrization of the Cornu spiral is by its tangent θ (1).

$$\begin{pmatrix} x(\theta) \\ y(\theta) \end{pmatrix} = a \begin{pmatrix} C(\theta) \\ S(\theta) \end{pmatrix} = \frac{a}{\sqrt{2\pi}} \int_0^\theta \begin{pmatrix} \frac{\cos(u)}{\sqrt{u}} \\ \frac{\sin(u)}{\sqrt{u}} \end{pmatrix} du \qquad (1)$$

The tangent θ is a function of the Cornu parameter t, $\theta = \pi t^2/2$. The term C and S are the Fresnel integrals and a is the scaling factor. The arc length of the Cornu spiral at the tangent value θ is defined as $s(\theta) = a\sqrt{2\theta/\pi}$ and the curvature $\kappa(\theta) = 1 / \mathrm{r}(\theta) = \sqrt{2\pi\theta}/a$. Fig. 2 shows a unit Cornu spiral with $a = 1$, interpolated from $0 \leq \theta \leq 4\pi$.

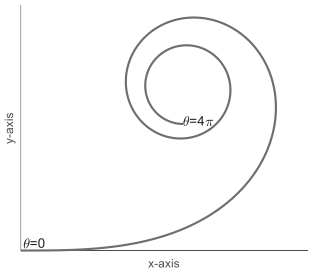

Fig. 2. unit Cornu spiral.

2.1. Solving Fresnel integrals under real-time constraint

Computing Fresnel integrals requires the use of numerical methods. One possible solution is to use Taylor expansions to approximate the sine and cosine function.

$$C(\theta) = \frac{1}{\sqrt{2\pi}} \sum_{n=0}^{\infty} \int_0^\theta (-1)^n \frac{\left(\frac{u}{\sqrt{u}}\right)^{2n+1}}{(2n+1)!} du$$

$$S(\theta) = \frac{1}{\sqrt{2\pi}} \sum_{n=0}^{\infty} \int_0^\theta (-1)^n \frac{\left(\frac{u}{\sqrt{u}}\right)^{2n+1}}{(2n)!} du$$

In [4], the modification of the Taylor expansions which eliminates the factorial to accelerate the computation time is described. To determine the suitability of the Taylor expansion, a test under a hard real-time operating system INtime tenAsys is conducted.

As can be taken from Table 1, Taylor expansions are suitable for a small interpolation range $\theta \leq 1.208\pi$, in which case only a small number of Taylor terms, $n = 11$, is required to obtain an error less than 1.1×10^{-9}. Since blending tasks commonly only require a small interpolation range $\theta \leq \pi$, Taylor expansions are suitable to solve the Fresnel integrals.

For higher interpolation ranges, either the integrals cannot be solved in real-time due to a high number of required Taylor terms, or the solution does not converge. An alternative approach based on rational approximation has therefore been developed for higher interpolation ranges [4]. Using the algorithm described in [4], the computation of Fresnel integrals for interpolation range $\theta \leq 5000\pi$ converges after ~120µs.

Table 1. Computation time for solving Fresnel integrals with Taylor expansion

Constraints	Case 1	Case 2	Case 3	Case 4
θ Start	0	0	0	0
θ End	25	15	0.88	4
Resolution	1000	1000	1000	1000
Taylor terms	37	37	11	11
Comp. time (~)	470µs	470µs	24µs	24µs

3. Algorithm for linear blending

For curved segments with inconstant curvature, the feedrate has to be computed depending on the curvature in order to maintain a constant MRR. The velocity variation, which in the long run may affect the drive motors negatively, can be avoided by inserting an additional arc segment with constant curvature between the Cornu spirals. This strategy is applied so that the blending curve is composed of Cornu spiral-arc-Cornu spiral. For the development of the algorithm, two types of blending are considered, symmetrical and asymmetrical corner-based blending, as well as intermediate point based blending (Fig. 3).

In case of corner-based blending an entry and exit point for the blending curve are given. Depending on the given points, this results in a symmetric or asymmetric segmentation. For the symmetric case, only one blending curve is computed and the second curve is obtained by reflection. For the asymmetric case, the Cornu spirals need to be computed separately. In case of intermediate point blending, an intermediate point is given and required to lie on the bisector so that by symmetry only one blending curve is required to be computed.

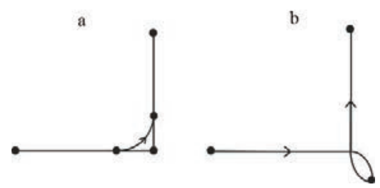

Fig. 3. (a) corner-based blending; (b) intermediate point blending.

3.1. Corner-based blending (symmetric)

In this case, the entry and exit point for the blending curves, as well as the angle β made by the lines are given (Fig. 4). Additionally, one property of the arc needs to be defined for increasing the degree of freedom. In [2], the radius r of the arc is given and the tangent angle θ is numerically solved. In our approach, the tangent angle θ is given, in order to solve the radius r analytically for minimizing computing time. Analogous to [2], the radius r can be obtained by defining the equation of the circle center $M(n, l)$, which due to symmetry lies at the bisector.

$$n = a\,S(\theta) + r\cos\theta \qquad (2)$$