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3D Tolerance Analysis with manufacturing signature and operating conditions

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Abstract

The present work shows a method to integrate the manufacturing signature and the operating conditions into a model for 3D tolerance analysis of rigid parts. The paper presents an easy way to manage the actual surfaces due to a manufacturing process and the operating conditions, such as gravity and friction, inside the variational model for a 3D tolerance analysis. The used 3D case study is deliberately simple in order to develop a conceptual demonstration.
The obtained results have been compared with those due to a geometrical model that reproduces what happens during assembly. It has been considered as reference case.

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1. Introduction

Tolerance analysis is a critical step to design and to manufacture a product. In fact, the need to assign dimensional and geometric tolerances to assembly components is due to the standardization of production and to the correct function of the assembly. The appropriate allocation of tolerances among the different parts of an assembly can result in lower costs per assembly and higher probability of fit, reducing the number of rejected parts or the amount of rework required on components.

Practically, dimensions and tolerances of assembly components combine, according to the assembly sequences, and generate the tolerance stack-up functions. Solving a tolerance stack-up function means to determine the nominal value and the tolerance range of a product function by combining the nominal values and the tolerance ranges assigned to the assembly components.

Many approaches for tolerance analysis exist in literature for rigid assemblies [1-2], but no one of them is completely and univocally accepted. Moreover, they reduce geometric deviations to translational and rotational part feature defects, and therefore they neglect form deviation [3-5].

In contrast to that, Samper et al. take into account form deviations of planar features in the computation of assemblies [6]. The approach is based on the modal description of form defects and the simulation results depend on the approximation of form deviations by eigenmodes. This limitation is overcome in the approaches by Stoll et al. [7,8], which are based on surface registration techniques.

However, these approaches can only handle discrete geometry representation schemes, which are not able to simulate the assembly behavior of variant parts based on their point cloud representation. Since point clouds are commonly obtained by assembly and measurement applications, their consideration in CAT tools is highly desirable to enable the connection among design, manufacturing and inspection. It has been developed a skin model inspired framework for the tolerance analysis [9, 10], which is based on a representation of non-ideal workpieces employing, such as point clouds. A further work uses Legendre-Fourier polynomials to model cylindricity error into a Jacobian-Torsor model for tolerance analysis [11].

In a previous work the authors developed a geometric approach to take into account form deviation, together with

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those due to location and orientation, in such a way to satisfy the Geometric Product Specification standards [12].

In the following, two approaches for the relative positioning of discrete geometry skin model shape for the application in computer-aided tolerance analysis are introduced, discussed and compared. The main contributions can be found in the development of a skin model shape based on a manufacturing signature, i.e. a systematic pattern that characterizes all the features manufactured by a process, and in the creation of an approach for assembly simulation of point-cloud skin model shapes taking into account gravity and friction.

The first approach consists of a geometrical tolerance analysis that takes into account both the manufacturing signature and the operating conditions, gravity and friction, during the assembling. It should numerically reproduce what happens in the actual assembling and, therefore, it represents the reference case.

The second approach is based on the variational model, that has been introduced and developed by Martino and Gabriele [13], Boyer and Stewart [14] and Gupta and Turner [15]. The idea is to represent the variability of an assembly, due to tolerances and assembly constraints, through a parametric mathematical model. It represents the dimensional and geometrical variations affecting a part by means of differential homogeneous transformation matrices. The variational model considers dimensional and geometrical tolerances applied to some critical points (contact points among profiles belonging to coupled parts) on the surface of the assembly components. These points are generally considered uncorrelated, since the ideal surface is taken into account.

In a previous work these two approaches have been applied to a 2D case study [16]. In this paper these approaches are discussed and applied to a 3D case study in the field of computer-aided tolerance analysis.

The paper is organized as follow: in Sec. 2, the reference case study is presented. In Sec. 3, the geometrical approach with manufacturing signature and operating conditions is deeply described. In Sec. 4, the variational model with manufacturing signature and operating conditions is discussed. Finally, in Sec. 5, the results are compared and discussed.

2. 3D Case study

The considered case study is constituted by three components: a hollow box and two spheres (SPH_1 and SPH_2), as shown in Fig. 1. A dimensional and a geometrical tolerances are applied to each sphere, while the box is considered nominal. The aim is the measurement of the gap *g* between the upper sphere and the top side of the box as a function of the tolerances applied to each component.

Each sphere has been simulated by a set of evenly distributed points, i.e the skin model shape. The amplitude of this set is equal to 235.822, i.e. it corresponds to a value of zenit and azimut angular steps equal to 0,45°, since it seems to be sufficiently large to simulate the assembling without slowing down too much the simulation. To each point of the sphere it has been applied the following error model:

$$|\mathbf{P_i} - \mathbf{O}| = R + r + d \tag{1}$$

where $\mathbf{P_i}$ is the generic point of the circular profile, $\mathbf{O}$ is the centre of the circle, *R* is the nominal value of the radius of the sphere (equal to 20 mm), *r* is the value due to the dimensional tolerance (equal to 0,0145 mm) applied to each sphere, and *d* is the value due to the manufacturing signature that should keep inside the form tolerance (equal to 0,0145 mm) applied to the spheres.

The *r* parameter has a Gaussian density function with mean value equal to zero and standard deviation equal to a sixth of the dimensional tolerance range.

The manufacturing signature on each sphere has been represented by means of a Simultaneous Autoregressive Model of first order SAR(1). This model has been chosen because it is suitable to simulate phenomena that are spatially correlated in more than one dimension. Traditional time series models, as the ARMAX model adopted in the 2D case [13], can represent correlation only along a single direction. The SAR(1) model instead can consider the spatial structure of the lattice defined by the triangulation of the points on the surface of the sphere at their nominal coordinates to generate a spatially correlated set of deviations from perfect sphericity. The SAR(1) model has been considered, rather than other higher order models in this first application, because it is easy to build and suitable to simulate deviations on a finite number of points.

In a SAR(1) model the deviations from perfect sphericity are simulated by means of the following equation:

$$d = (\mathbf{I} - \mathbf{G})^{-1}\boldsymbol{\varepsilon} \tag{2}$$

where $\mathbf{I}$ is the identity matrix, $\mathbf{G}$ is a weight matrix and $\boldsymbol{\varepsilon} \sim N(0, \sigma^2\mathbf{I})$ is a white noise. σ is equal to 0,0024 mm.
In particular, $\mathbf{G} = \rho\mathbf{W}$. ρ is a correlation coefficient. Higher values of ρ denote a higher degree of spatial correlation among nearby points. Its value is 0,9. $\mathbf{W}$ is a neighbourhood matrix defined based on the triangulation of the points on the surface of the sphere. In particular,

$$w_{ij} = \frac{\frac{I_{ij}}{d_{ij}}}{\sum_k \frac{I_{kj}}{d_{kj}}} \tag{3}$$

in which d_{ij} is the Cartesian distance between the $\mathbf{P_i}$ and the $\mathbf{P_j}$ points of the sphere, and I_{ij} is an indicator variable, which denotes whether points i and j are neighbours, that is

$$I_{ij} = \begin{cases} 1, \text{if point } i \text{ and } j \text{ belong to a same triangle} \\ 0, \text{otherwise} \end{cases} \tag{4}$$

An example of a sphere is shown in Fig. 2.