Available online at www.sciencedirect.com
ScienceDirect
Procedia CIRP 43 (2016) 214 – 219
www.elsevier.com/locate/procedia

14th CIRP Conference on Computer Aided Tolerancing (CAT)

A hierarchical category model for geometrical product specifications (GPS)

Qunfen Qi[a,*], Paul J Scott[a], Xiangqian Jiang[a], Wenlong Lu[b]

[a]*EPSRC Centre for Innovative Manufacturing in Advanced Metrology, Centre for Precision Technologies, School of Computing and Engineering, University of Huddersfield, Huddersfield, HD1 3DH,UK*

[b]*The State Key Laboratory of Digital Manufacturing Equipment and Technology, School of Mechanical Science and Engineering, Huazhong University of Science and Technology, Wuhan 430074, PR China*

* Corresponding author. Tel.: +44-1484-471284; fax: +44-1484-472161. *E-mail address:* q.qi@hud.ac.uk

Abstract

International standards for tolerancing (ISO GPS) have undergone considerable evolutionary changes to meet the demands of the modern information age. Their expanding quantity and complexity have proposed a great obstacle to their informatisation progress. In this paper, a solution to reduce the complexity is coarse-graining the GPS knowledge into five hierarchy levels. A high-level abstraction mathematical theory – category theory is employed to model the GPS hierarchy, in which structures are modelled by categorical concepts such as categories, morphisms, pullbacks, functors and adjoint functors. As category theory is hierarchically structured itself, it can prove that the multi-level GPS framework is constructed in a rigorous manner and is expected to facilitate the future autonomous integration between design and measurement in the manufacturing system.

Peer-review under responsibility of the organizing committee of the 14th CIRP Conference on Computer Aided Tolerancing
Keywords: Geometrical Product Specifications (GPS); category theory; hierarchy; coarse-graining

1. Introduction

Over the last decade, International standards for tolerancing (ISO GPS) have undergone considerable evolutionary changes to meet the demands of the modern information age [1,2]. The standard system is expanding on both quantity and complexity, which have proposed a great obstacle to its informatisation progress [3,4]. There have been continuing efforts that directed toward developing knowledge models of ISO GPS [5–10], as well as incorporating GPS information into computer-aided systems [2,11]. Yet there is no comprehensive tool/model that naturally support the structural knowledge of GPS and enriched GPS concepts and semantics.

In the GPS system the interactions between design (specification) and measurement (verification) are dual. An inspector measures the surface with guidance from technical drawing/symbols. The observed (measured) data can only be considered meaningful if it can be interpreted in the range of theoretical model. When the meaningfulness of the observed data is proved the conformance process can then be taken place. The two stable mappings between the specification and verification serve the structure of adjoint functors in category theory, a high-level abstraction mathematical theory which was invented by Samuel Eilenberg and Saunders Mac lane in 1942-1945 [12]. The concept of adjoint functors is seen as central to category theory. Some category theorists consider adjoint functors as dictionaries that translate back and forth between categories[13]. If the two categories are two languages (say English and Chinese) which are equally expressive, then a good dictionary will be an explicit exchange of ideas. Employing the adjoint functors and other structures of category theory to translate specification information into verification and vice versa has the great potential to bring the ISO GPS system toward an autonomous manner.

From 1980s to the present, we have seen many successful category-theoretical applications in theoretical computer science, theoretical physics and biological. Researchers are using category theory to study complex systems [14], cognitive neural networks [15,16], biological networks [17] and model management [18]. Category theory has also been employed for the framework of knowledge representation in relational [13] and object-oriented styles [9,10]. Using object-oriented language to code the categorical model has recently been proved successful in the case of surface texture [19], one of the most complicated geometrical specification and verification systems in GPS.

In this paper, using the categorical model, structural knowledge of GPS is coarse-grained into five hierarchy levels, which is expected to reduce the complexity of the design and measure-

Peer-review under responsibility of the organizing committee of the 14th CIRP Conference on Computer Aided Tolerancing
doi:10.1016/j.procir.2016.02.165

ment, guarantee their stability and traceability. In this approach, the measurement process is modelled by a top-down approach (from the highest hierarchy to the lowest). The designing process of specification elements can then be conducted using a bottom-up approach. Adjoint functor is utilised in the categorical model to ensure the two mappings between specification and verification are structure-preserving and stable.

The paper is constructed as follows. Basic knowledge of category theory including adjoint functors is introduced in section 2. Five levels of hierarchy category model are structured in section 3. How the hierarchical model can be applied for the automation of specification and verification has been discussed in section 4. Section 5 summaries the paper.

2. A brief of category theory

Category theory (CT) itself is hierarchically structured which can be summarised into three levels as shown in Fig.1.

A category is construed as a collection of 'things' and a type of relationship between pairs of such 'things' [20]. The 'things' are called objects and the 'relationships' are called morphism in the category.

Definition 2.1 A category $\mathcal{C}$ consists of a collection of objects $A, B, C...$, denoted as $Ob(\mathcal{C})$; for every pair of objects $A, B \in Ob(\mathcal{C})$, a set $Hom_{\mathcal{C}}(A, B)$ is called the *hom-set* from A to B; its elements called morphisms from A to B, and satisfy the identity law and associativity law.

The universal constructs (middle part of Fig. 1) are objects and morphsims. It is also including operations between objects within a category, such as product and coproduct.

The lower order includes the properties of universal constructs, which includes domain, codomain, epic, monic, isomorphic, initial objects, terminal object etc. An object I is said to be initial if for every other object X there is exactly one morphism $f : I \to X$. An terminal object T is that for every other object X there is exactly one morphism $f : T \to X$. More details of those properties can be found in Refs[13,21,22].

In the higher order, a set of objects constructs a category, morphsims between categories are functors, morphsims between functors are natural transformations, and if there is a functor has an inverse functor, the pair is called adjoint functor.

Definition 2.2 Let $\mathcal{C}$ and $\mathcal{D}$ be categories. An adjunction between $\mathcal{C}$ and $\mathcal{D}$ consists of two functors $F : \mathcal{C} \to \mathcal{D}$ and $F^+ : \mathcal{C} \to \mathcal{D}$, and a natural isomorphism whose component for any objects $D \in Ob(\mathcal{D})$ and $C \in Ob(\mathcal{C})$ is:

$$\eta_{\mathcal{C},\mathcal{D}} : Hom_{\mathcal{D}}(F(C), D) \cong Hom_{\mathcal{C}}(C, F^+(C))$$

This isomorphism is called the adjunction isomorphism for the (F, F^+) adjunction, and for any morphism $f : F(C) \to D$ in $\mathcal{D}$, we refer to $\eta_{\mathcal{C},\mathcal{D}}(f) : C \to F^+(D)$ as the adjunct of f.

The functor F is called the left adjoint and the functor F^+ is called the right adjoint. $\mathcal{C}$ might be called the sending category and $\mathcal{D}$ the receiving category. This setup often denote by

$$F : \mathcal{C} \rightleftarrows \mathcal{D} : F^+$$

Amongst concepts in CT, adjoint functor is seen as central. We often have two categories that are not on the same conceptual world, and the adjoint functors connect two different structures by structure-preserving mapping. That is why adjoint functors often come in the form of 'free' and 'forgetful'. One particular example is a forgetful functor which is defined from a category of algebraic structures (group or vector spaces) to the category of sets. The forgetful functor forges the arrows, remembering only the underlying set and regardless of their algebraic properties.

3. Categorical modelling schema - a hierarchy structure

Theoretically speaking, the GPS system is structured by geometrical features which defined by geometrical operations. All geometrical features can be classified into three invariance types: simple class, generated class and complex class (freeform), and each of which has different types of features.

The operations that define these features can be summarised by a pair of operations: decomposition and composition. Decomposition is an operation that decomposes a surface into different features, and composition is an operation that builds a surface up from different features. The two operations can be decomposed into an ordered set of operations, which can be refined into elements of operations that can still be gradually detailed into different levels.

Therefore the behaviour of the system can be resolved at multiple scales and the interactions at different scales inform each other. There are two ways that this information can be propagated. Top-down: the behaviour at larger scales is used to inform the interactions at more detailed scales. Bottom-up: information at smaller scales is used to inform models at larger scales.

Thereby in this section, a hierarchy structure of GPS is developed using categorical modelling schema. Five levels of the hierarchy are modelled using CT with reference to the structure of features and operations.

3.1. The Top level

The top level of the hierarchy is set to identify the specification features and the measurement features.

In the world of design, an artefact is presented by skin model which can be decomposed into surface features. This operation is also called 'partition', an operation that identifies bounded features such as point, straight line or plane. Specification features can then be defined by a series of decomposition and composition operations carried on the separated features, such as plane surface, a cylindrical surface or a prismatic surface.

From the introduction of CT, decomposition and composition can be view as a pair of adjoint functors. The two basic operations defined seven feature operations (defined by ISO TC 213), which are termed 'partition', 'extraction', 'filtration', 'association', 'collection', 'construction' and 'reconstruction' [23]. The set of ordered operations define the specification operator/operators for a specified feature. Note that there might be more than one operator for a specified feature. Specified features that are location, orientation or run-out are always with at least two or more operators as each of their required datum has related operator as well.

A specification feature from a skin model cannot be determined by the skin model itself. Assembly relationships between skin models and constraints between features in different skin models will be combined to determine the specified feature type. As shown in Fig.2, morphism A_1 and its inherited mor-