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Process Tolerancing: a new approach to better integrate the truth of the processes in tolerance analysis and synthesis

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Abstract

Limits of traditional Tolerancing methods are demonstrated for long. Worst-Case is so pessimistic and requires a 100% checking, providing scrap for nothing. Statistical Tolerancing RSS becomes risky when idealized centering assumptions are not perfectly achieved. New reliable methods exist, allowing to achieve the Capability requirement on resulting criteria, by using "population specifications" from ISO 18391.
One is "Inertial Tolerancing", from Pillet. We propose an alternative named "Process Tolerancing", improving Semi-Quadratic methods from Mansoor, Greenwood or Taylor, and better adapted to industries of not daily adjustable toolings. This paper compares these 2 methods and illustrates their differences.

Keywords: Statistical Tolerancing, Integrated Design, SPC

1. Introduction

In industry, the mission of designers is to define the product and the process in order to satisfy the Customer. Unfortunately, there is no process able to produce all a year the same identical parts or assembly. Tolerancing activities becomes necessary each time process variability may have an effect on customer satisfaction and must be managed.

Starting point is to identify the criteria linked to customer satisfaction, the Y's. Relating to product performances or failure modes, all Y's fulfil a common definition "*criteria for which one a non-conformity is a defect for the customer*".

It could be sufficient to check the conformity only on Y's. Unfortunately, the cost of a defect at end of line may be too high, and it is generally useful and profitable to operate an appropriate control upstream on causes at components or process level. In this perspective, Engineers analyze product and process functioning and identify components and process parameters requiring to be driven in order to achieve the desired performance. We name these parameters the X's, and a primary task is to determine the transfer function from the X's toward the Y's, that is to say, the Causes-Effect Relationship : $Y=F(X_i)$, i=1…N.

To drive the Xi means to specify and to control them, and a first question appears: *"How to specify the Xi to get the desired performance on the Y ?"*. Next question will be *"how to control them?"*. These two questions require coherent responses, but this coherence is not properly achieved with the most popular Tolerancing methods. New and reliable approaches are necessary and hopefully exist today.

2. Existing methods for Tolerances Analysis & Allocation

2.1. About linear approximation

Linear approximation validity domain covers a wide and attractive field in the industry. First, a functioning with significant nonlinearities or heavy interactions is never optimal for robustness, and Engineers naturally tend to avoid solutions with erratic behavior. Secondary, when X's variations are small, linear approximation provides an effective prediction of performance variations around the target. We consider here to be in this linear approximation validity domain, and we have:

$$Y \approx a_o + \sum a_i . X_i \qquad (1)$$

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The constant a_0 always allows to well fit the average, and the quality of the linear model can be assessed by the determination coefficient R² giving the part of the variance explained by the model. When the function F is nonlinear, a_i coefficients must be updated when a nominal value changes.

A model providing the right average and the right variance can be considered as statistically good, especially when common hypothesis of normal distribution for resulting criteria is generally going to be done.

Nomenclature

Y	Customer criteria with conformity requirement
t_Y	Target for resulting criteria Y
Tol_Y	Tolerance interval for resulting criteria Y
μ_Y	Process average for resulting criteria Y
σ_Y	Process standard deviation for resulting criteria Y
δ_Y	Average deviation to the target for Y : $\delta_Y = t_Y - \mu_Y$
Cp_Y	Potential Process Capability Index on Y
$CpkL_Y$	Lower Process Capability Index on Y
$CpkU_Y$	Upper Process Capability Index on Y
Cpk_Y	Minimum Process Capability Index on Y
X_i	Product or Process parameter having an impact on a Y
t_i	Target for contributor X_i
Tol_i	Tolerance interval for contributor X_i
μ_i	Process average for contributor X_i
σ_i	Process standard deviation for contributor X_i
δ_i	Average deviation to the target for X_i : $\delta_i = t_i - \mu_i$
Cp_i	Potential Process Capability Index on X_i
Cpk_i	Minimum Process Capability Index on X_i

2.2. Problem data

A customer criteria Y is generally specified by its conformity or specifications limit LsL and UsL with associated requirement on Capability Indexes $CpkL_Y$ and $CpkU_Y$. In case of Normal distribution for the Y, Capability Indexes are:

$$CpkL_Y = \frac{\mu_Y - LsL_Y}{3.\sigma_Y} \quad \& \quad CpkU_Y = \frac{UsL_Y - \mu_Y}{3.\sigma_Y} \qquad (2)$$

On the Xi, it is usual to specify a tolerance around a target t_i

$$X_i = t_i \pm \frac{Tol_i}{2} \qquad (3)$$

Then first question becomes: "How to allocate targets ti and tolerances Tol_i to achieve the Capability requirement on Ys.

We are going to analyze existing methods in the simplified case where the Xi criteria impact only one given Y.

2.3. Using Worst-Case Tolerancing

The most popular method for Tolerance Calculation consist on a simple stacking of the tolerances. The formulas are:

resulting target	*resulting tolerance*
$t_Y = a_o + \sum a_i.t_i$	$Tol_Y = \sum \lvert a_i \rvert.Tol_i$

$$\textit{And then} \qquad Y = t_Y \pm \frac{Tol_Y}{2} \qquad (4)$$

Tolerance allocation can be validated if

$$t_Y - \frac{1}{2}.Tol_Y \geq LsL_Y \quad \& \quad t_Y + \frac{1}{2}.Tol_Y \leq UsL_Y$$

Capability requirements on customer criteria Y has no influence on tolerance allocation on the Xs in this calculation. The only requisite is to get parts inside their tolerances to guarantee an assembly inside its specification. Many companies using Worst-Case impose to cascade the Cpk requirement from the Y to the Xi, and it is interesting to clarify the relationship between the Capability index on the Xi and the resulting Capability on the Y.

$$\text{From :} \qquad Cpk_Y = Cp_Y - \frac{|\delta_Y|}{3\sigma_Y} \qquad (5)$$

And considering all average deviations on Xi may have "at worst" an effect on the same bad side for Y, we obtain:

$$Cpk_Y \geq \sum \sqrt{Cv_i}.Cpk_i \quad \textit{with} \ Cv_i = \frac{a_i^2.\sigma_i^2}{\sigma_Y^2} \qquad (6)$$

Where Cvi est the contribution of Xi to the Y variance

We get: $\sum_i \sqrt{Cv_i}.Cpk_i \geq Cpkx_{\min}.\sum \sqrt{Cv_i}$

This demonstrates that Capability on Y is at least equal to the lower Capability index on Xi, because $\sum \sqrt{Cv_i} \geq 1$

This worst situation happens when one single contributor provides all the variance, and when the others, as a dirac's delta functions, are fully off-centered on a same bad side of their tolerances. They produce together the Go or NoGo gage for the first one and reveal its own fraction nonconforming.

We understand that the cascading of Cpk requirement from the Y to the Xs comes from the fear of an event that probably never happen.

In the case of a stack of N components Xi with identical contributions, and achieving their Cpk requirement, we have:

$$Cpk_Y \geq \sum_i \sqrt{\frac{1}{N}}.Cpk_i. = \sqrt{N}.Cpk_X \qquad (7)$$

So, in many case, to achieve a Cpk=1 on contributors from a stack of 3 and more components is enough to get a Cpk over 1.67 on the assembly. But if Go/nogo gages are used on some X, it becomes not enough to get a Cpk=1 on the others...

We propose another way to illustrate the pessimism of Worst-case method. When one X goes out its tolerance, it doesn't enter in "a wall" but inside the cumulated tolerance of the others. If the other X's are supposed to be uniformly distributed inside their tolerances, the cumulated distribution, calculated by convolution, becomes a belt curve converging gradually toward a Normal distribution when the number of contributors increase. Then the defect probability on the Y, resulting from this exit, doesn't switch abruptly from 0 to 100%, but increase along a S curve as shown on Fig.1.

This first figure is built on the case of 5 contributors, where an exit out tolerance for one X, about 73% of its tolerance range generate a risk less than 1% on the assembly. Fig.2 shows how this risk varies according to the number of contributors