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Machine stiffness rating: Characterization and evaluation in design stage

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Abstract

Stiffness of mechanical systems of machines with heavy demands on their accuracy, precision and productivity such as machine tools, coordinate measuring machines, and industrial robots presents one of the most important design criteria. However, stiffness evaluation in the general case when force-and-torque load coupling takes place leads to some problems. The problems are associated with physical distinction between translational and rotational stiffness values manifested, in particular, in their different units of measurement. To overcome a majority of the difficulties, a new performance index – the collinear stiffness value (CSV) presenting an equivalent stiffness (compliance) value during simultaneous linear and rotational displacements – is developed and represented in static and dynamic versions. In this presentation, the CSV is used to formulate a new design-related dimensionless criterion: the ratio of the minimal CSV to stiffness value of the drive system, which usually presents a weak point of the modern machines. The CSV-based approach is applied to quantitative formulation of the significant advantage from the stiffness viewpoint of the orthogonal serial-kinematics machines (SKM) compared with the parallel-kinematics machines (PKM): (a) the parameters-of-motion-depending variations of the minimal CSV of the SKM in their workspace are, as a rule, one-two orders of magnitude less than those of the PKMs; (b) the stiffness-limited workspace of the SKM is more than that of the PKM. Application examples are simulated.

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1. Introduction

In recent years, the manufacturing industry has seen a rise in the demands for accuracy, precision and increased productivity of machined components. To achieve the needed precision, machine tool manufacturer should identify the most significant characteristics defining the machine performances and apply multi-variable optimization techniques during the design process. Stiffness, the capacity of mechanical system to sustain loads without excessive changes of it geometry (deformations) [1], it is one of the most important design criteria for mechanical components, systems and machine tools. The stiffness has a direct impact on the position accuracy and presents one of the key parameters used for comparison of machine tools with different kinematic types [2,3–5]. In many structures as machine tools, there are critical directions along which deflections must be minimal (i.e., stiffness must be maximized). In this context, the stiffness performance indexes are intensively investigated [7–8]. Since the elements of the stiffness matrix are of heterogeneous units of measurement, there are difficulties in application of a

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matrix trace, non-zero eigenvalues, and condition numbers to direct evaluation of the structural stiffness. To overcome the heterogeneity problem a new performance index – the collinear stiffness value (CSV) – is developed and represented in static and dynamic versions [1, 2]. The CSV has natural units of measurement and takes the value 0 in the cases (and only in such cases), in which the regular defined stiffness value is equal to zero: in singular configurations of the parallel-kinematics machines in statics and in any configuration when its vibration frequency becomes the natural one. The CSV is configuration-dependent, and its minimal CSV allows limitation of the workspace according to stiffness requirements and construction of virtual barriers preventing approaching the configurations with non-tolerable stiffness.

In this paper, a new design-related performance index using the CSV-based approach is formulated and applied to stiffness evaluation in the machine workspace: the ratio of the *minimal* CSV to a stiffness value of the motion-actuating drive. As is known from the modern design practice, the drive usually presents a weak point of the mechanical system from the stiffness viewpoint [1]. This ratio named "stiffness rating" allows quantitative evaluation of the following important design parameters: (i) decreasing the structure stiffness in the machine workspace as compared with its weak element; and (ii) variation of the stiffness rating in the machine workspace caused by functional motions of machine units. As a new important application, the CSV-based approach is used for quantitative comparison of the machines (SKM) with different kinematics according to absolute values of stiffness parameters and their variation in the machine workspace. Application examples are simulated.

2. Collinear stiffness value: Definitions and formulations

2.1. General definitions

When a machine tool performs a given task the tool exerts force and/or moment on the workpiece and on the machine tool structure. This contact force and/or moment will cause the tool to deflect away from its desired planed location depend on the machine tool structural stiffness. The overall stiffness of a machine tool depends on several factors, including the machine elements configuration, material, the mechanical transmission mechanisms, the slides, and the controller. The information about stiffness of the machine in a given configuration consists of the 6×6 Cartesian stiffness matrix **K** connecting static wrench **f** and twist **δ**,

$$\mathbf{f} = \mathbf{K}\,\boldsymbol{\delta} \tag{1}$$

where $\mathbf{f} = [P_x, P_y, P_z, T_x, T_y, T_z]^T$ is the 6×1 vector of active forces and torques acting along the *X*-, *Y*-, *Z*-axis (in N and N-m/rad, respectively); and $\boldsymbol{\delta} = [\delta_x, \delta_y, \delta_z, \delta_\theta, \delta_\psi, \delta_\varphi]^T$ is the 6×1 vector of magnitudes of linear and angular displacements of a body (in m and rad., respectively).

The traditional definitions of stiffness values are defined for two partial cases of body displacements: either pure translation or pure rotation [1,2,5,6]. In the former case, the stiffness value presents the force induced by unit linear displacement, and in the latter case, it is the torque induced by unit angular displacement. However, in the general case, in which the body experiences both force and torques, the traditional definition of the stiffness value is inapplicable since the Euclidian norm of the vector **δ** has no physical meaning because of the heterogeneity of its units of measurement. To evaluate the stiffness value in the general case, when a body after being subjected to both angular and translational displacements, the collinear stiffness value (CSV) is proposed [7,8].

$$k_{col} = \mathbf{s}^T\,\mathbf{K}\,\mathbf{s} \tag{2}$$

where **K** is the stiffness matrix, Eq. (1), and **s** is the unit screw [6], defined through the screw directional cosines c_x, c_y, c_z with $|c_x, c_y, c_z| = 1$ and translations n_x, n_y, n_z; screw **s** has two forms – the unit rotational screw $\mathbf{s}_r$ and the unit rotational screw $\mathbf{s}_t$ - depending on values of angular components δ_θ, δ_ψ, δ_φ, entering in the screw **δ**, Eq. (1),

$$\mathbf{s}_r = [n_x, n_y, n_z, c_x, c_y, c_z]^T, \text{ for } \|[\delta_\theta, \delta_\psi, \delta_\varphi]\| \neq 0 \tag{3}$$
$$\mathbf{s}_t = [c_x, c_y, c_z, 0, 0, 0]^T, \text{ for } \|[\delta_\theta, \delta_\psi, \delta_\varphi]\| = 0 \tag{4}$$

The CSV k_{col}, Eq. (2), presents the natural evaluation of the stiffness value in any *given configuration*, both regular and singular, associated with the stiffness matrix **K**. It is the minimal value of the CSV that is of primary concern to engineering design. Since the CSV is defined in two forms, Eqs. (3) and (4), the minimal values are also defined in two forms – minimal rotational CSV $(k_r)_{min}$ and minimal translational CSV $(k_t)_{min}$:

$$(k_r)_{min} = \min\,(\mathbf{s}_r^T\,\mathbf{K}\,\mathbf{s}_r) \tag{5}$$
$$(k_t)_{min} = \min\,(\mathbf{s}_t^T\,\mathbf{K}\,\mathbf{s}_t) \tag{6}$$

where minimum is found out for all possible values n_x, n_y, n_z, c_x, c_y, c_z associated with the given configuration.

Hence, there are two minimal stiffness values associated with each machine configuration. A procedure for calculation of the minimal values $(k_r)_{min}$ and $(k_t)_{min}$ use the Lagrange multiplier method [7,8].

In terms of the minimal CSV, Eqs. (5) and (6), a dimensionless design-related performance index can be formulated and applied to stiffness evaluation in the machine workspace: the ratio of the *minimal* CSV to a stiffness value of the rotation-actuating drive k_{rot} and translation-actuating drive k_{tr},

$$\xi = (k_r)_{min}/k_{tr} \tag{7}$$
$$\zeta = (k_t)_{min}/k_{rot} \tag{8}$$

These ratios named a *stiffness rating*. Choosing the drive stiffness values as a reference point for stiffness rating is explained derives from two evident facts: (a) the drive makes an integral part of a machine, and (b) as is known from the modern design practice, the drive usually presents a weak point of the serial-type mechanical