



An elementary model for the transition from conductive to penetrative burning in gas-permeable explosives

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ABSTRACT

The experimentally known phenomenon of spontaneous transition from slow conductive to fast penetrative (convective) burning in a confined gas-permeable explosive is discussed. A reduced quasi-linear model, involving only the most essential physical ingredients, is formulated. A good qualitative agreement between theoretical and experimental dependencies is obtained. Similar to the previously studied case of an inert porous matrix filled with an explosive gas, the transition is triggered by a localized autoignition in the extended resistance-induced preheat zone gradually formed ahead of the advancing deflagration. The conventional concept of penetrative burning is re-examined.

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1. Introduction

A self-sustained wave of the exothermic chemical reaction spreading through a homogeneous gaseous explosive is known to occur either as a subsonic deflagration (driven by thermal conductivity) or by supersonic detonation (driven by adiabatic compression). In unconfined obstacle-free systems the concrete realization of the specific propagation mode is controlled by the ignition conditions. Normally, deflagrations are initiated by a mild energy discharge, e.g. by a spark, while detonations are provoked by shock waves via localized explosion. It is known however that in the presence of obstacles or confinement (tube walls, porous matrix) the initially formed deflagration undergoes slow acceleration, ending up abruptly as a detonation. Apart from inducing hydrodynamic disturbances, and thereby affecting the deflagration wave speed, the obstacles also exert resistance to the gas flow causing reduction of its momentum and local elevation of the pressure. The gradual pileup of the pressure results in the formation of an extended preheat zone ahead of the advancing flame. This slowly proceeding development may ultimately end up as an adiabatic explosion which abruptly converts the burning process from deflagration to detonation [1,2].

The outlined resistance-based concept of the deflagration-to-detonation transition in gaseous systems is likely to be of relevance also for understanding the transition from slow to fast burning occurring in confined gas-permeable solid explosives/propellants. As soon as the incipient flame penetrates into the interior of the explosive, pressure gradients develop due to the resistance to the gas

flow in the interstices between the explosive particles. One thus ends up with a situation very similar to that occurring in gaseous systems.

Although the importance of hydraulic resistance for the transition in gas-permeable explosives has long been recognized [3–13], its specific impact, to our knowledge, has not yet been fully understood. In particular, the mechanism responsible for the abrupt transition from slow conductive to fast penetrative (convective) burning has not been properly identified. Isolation of the transition mechanism is the main objective of this paper.

2. Formulation

For solid gas-permeable explosives the penetrative burning is not, as a rule, a final equilibrium state but rather a transient stage toward the ultimate supersonic detonation, involving compaction of the explosive, collapse of its permeability, and compression of the full-density (impermeable) explosive by the advancing shock. To isolate the mechanism controlling the transition from conductive to penetrative burning the effect of compaction may be provisionally ignored, as being of a different physical nature. As a result, upon the transition the penetrative burning may become an equilibrium state. Experimentally, some evidence of steady penetrative burning has been observed by several investigators [14–16]. The theoretical possibility of constant-speed penetrative burning has long been shown by Kuo and Summerfield [4].

To simplify further analysis we assume that the explosive is foam-like; that is, highly energetic with the porosity close to unity. One therefore may consider the limit where the gas generation is negligible but the effect of heat release is substantial. The continuity equation for the gaseous phase may then be written without the mass-generation term. Practically, a high gasification rate is the

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principal function of many explosives. Yet, curiously enough, this aspect is not crucial for the transition event and in the qualitative physical analysis may well be ignored. For the same reason we keep the conventional no-slip boundary condition for the gas flow within the porous matrix and employ the classical Darcy's law as the momentum equation. As the equation of state a perfect gas law is used. The gas and solid are assumed to be in thermal equilibrium, allowing utilization of a single-temperature description for the two-phase medium. The reaction rate is specified as a one-step zero-order Arrhenius kinetics converting the solid explosive straightforwardly into the gaseous product. Upon its consumption the solid explosive ceases to exert resistance to the gas flow—the permeability of the system ascends to infinity. The current study is focused on the subsonic propagation only. Moreover, we consider the limit of zero-Mach-number, where the effects due to dynamic compressibility are ignored. In this case the pressure in the drag-free burned gas region becomes spatially uniform, though time-dependent.

Apart from the neglect of gas production and volume-fraction of the solid phase, the above premises are similar to those adopted by Margolis in his ground-breaking theoretical exploration of the convection-enhanced burning [10] (for its further developments see Refs. [11–13]).

In the high porosity limit ($\phi \simeq 1$) the effective material properties of the gas–solid system are determined by the gaseous phase only. The function of the solid phase is merely to provide the source of energy and to induce hydraulic resistance. The set of governing equations may thus be written as follows.

Mass conservation for gaseous phase,

$$\rho_t + (\rho u)_x = 0. \quad (1)$$

Momentum conservation (Darcy's law),

$$\rho u = -(\mathcal{K}/\nu)P_x. \quad (2)$$

Energy conservation,

$$c_p \rho T_t + c_p \rho u T_x = c_p (\rho D_{th} T_x)_x + P_t + (1 - \phi)QW. \quad (3)$$

Equation of state,

$$P = (c_p - c_v)\rho T. \quad (4)$$

Solid reactant conservation,

$$\rho_s C_t = -W. \quad (5)$$

Reaction rate,

$$W = AH(C) \exp(-T_a/T). \quad (6)$$

Here T , P , C , ρ , u are the temperature, pressure, solid reactant mass-fraction and flow velocity, respectively. ρ_s is the density of the solid phase, c_p , c_v are specific heats, D_{th} , ν are thermal diffusivity of the gaseous phase and its kinematic viscosity—all assumed to be constant. A is the pre-exponent. T_a is the activation temperature. $H(C)$ is the Heaviside step-function,

$$H(C) = 1 \text{ at } C > 0 \text{ and } H(C) = 0 \text{ at } C = 0. \quad (7)$$

\mathcal{K} is the permeability of the system, dependent on the reactant mass-fraction, C . For simplicity we specify this dependency as

$$\mathcal{K} = \mathcal{K}_0(H(C))^{-1}. \quad (8)$$

That is, according to Eq. (2), $\rho u = -(\mathcal{K}_0/\nu)P_x$ at $C > 0$, and $P_x = 0$ at $C = 0$. ϕ is the porosity of the system, which (in line with the permeability) is specified as

$$\phi = 1 + (\phi_0 - 1)H(C). \quad (9)$$

That is, $\phi = \phi_0$ at $C > 0$, and $\phi = 1$ at $C = 0$.

As an additional simplification we adopt the quasi-linear (small-heat-release) approximation where variations of temperature, pressure and flow velocity are regarded as small and, hence, the nonlinear effects are ignored everywhere but in the reaction-rate and permeability terms, highly sensitive to even minor changes in temperature and the reactant's mass-fraction. In this formulation the problem becomes much more compact mathematically, without seemingly compromising crucial features of the original fully nonlinear system. Thus we set, $P = P_0 + \delta P$, $T = T_0 + \delta T$, $\rho = \rho_0 + \delta \rho$, $u = \delta u$, where P_0 , T_0 , ρ_0 correspond to the initial state of the system prior to ignition. Upon partial linearization, assuming δ -terms to be small, Eqs. (1)–(6) yield

$$(\delta \rho)_t + \rho_0(\delta u)_x = 0, \quad (10)$$

$$\rho_0(\delta u)_t = -(\mathcal{K}/\nu)(\delta P)_x, \quad (11)$$

$$c_p \rho_0(\delta T)_t = c_p \rho_0 D_{th}(\delta T)_{xx} + (\delta P)_t + (1 - \phi)QW, \quad (12)$$

$$(\delta P) = (c_p - c_v)\rho_0(\delta T) + (c_p - c_v)T_0(\delta \rho), \quad (13)$$

$$\rho_s C_t = -W, \quad (14)$$

$$W = AH(C) \exp(-T_a/(T_0 + \delta T)). \quad (15)$$

Here \mathcal{K} and ϕ are defined by Eqs. (8), (9).

3. Nondimensionalization

For further discussion we introduce nondimensional variables and parameters defined as

$\Theta = (\delta T)/(T_\infty - T_0)$ —scaled temperature,

$\Pi = (\delta P)/(P_\infty - P_0)$ —scaled pressure,

$\Sigma = P_0(\delta \rho)/\rho_0(P_\infty - P_0)$ —scaled density,

$U = (\delta u/u_\infty)$ —scaled flow velocity,

$\Phi = C/C_\infty$ —scaled solid reactant mass-fraction,

$\Omega = W/W_\infty$ —scaled reaction rate,

$\xi = x/x_\infty$, $\tau = t/t_\infty$ —scaled spatio-temporal coordinates,

$\kappa = \mathcal{K}/\mathcal{K}_\infty$ —scaled permeability.

Here, $T_\infty = T_0 + (1 - \phi_0)Q\rho_s C_0/c_v \rho_0$ and $P_\infty = P_0(T_\infty/T_0)$ are the final temperature and pressure of combustion products reached upon the adiabatic homogeneous explosion. C_0 is the reactant mass-fraction prior to ignition. $W_\infty = (1 - \gamma^{-1})^{-1}A \exp(-T_a/T_+)$, where $\gamma = c_p/c_v$, and T_+ is the 'ignition' temperature defined by the relation, $T_\infty - T_+ = (1 - \phi_0)Q\rho_s C_0/c_p \rho_0$, or $T_+ = T_0 + (1 - \gamma^{-1})(T_\infty - T_0)$.

The normalizing factor $(1 - \gamma^{-1})^{-1}$ in W_∞ keeps the scaled velocity of the well-settled penetrative burning near unity (see Fig. 2 and Eq. (40) below). $t_\infty = \rho_s C_0/W_\infty$, $x_\infty = \sqrt{D_p t_\infty}$, $u_\infty = x_\infty/t_\infty$, where $D_p = \mathcal{K}_0 a_0^2/\gamma$ is the pressure diffusivity, and $a_0 = \sqrt{\gamma(c_p - c_v)T_0}$ is the speed of sound. $\beta = (1 - \sigma)(T_a/T_+)$ is the Zeldovich number based on T_+ with $\sigma = T_0/T_\infty$. $\epsilon = D_{th}/D_p$ is the ratio of thermal and pressure diffusivities. ϵ is often rather a small number ensuring a marked disparity between the propagation velocities of conductive and penetrative modes. For example, for a set of typical values: $\gamma = 1.4$, $a_0 = 350 \text{ ms}^{-1}$, $\nu = D_{th} = 2.5 \times 10^{-5} \text{ m}^2 \text{ s}^{-1}$, $\mathcal{K}_0 = 2.5 \times 10^{-10} \text{ m}^2$, one obtains $\epsilon \simeq 10^{-5}$.

Introducing the above nondimensionalizations into Eqs. (10)–(15) one ends up with the following set of scaled equations:

$$\Pi_\tau - \Theta_\tau + U_\xi = 0, \quad (\Pi - \Theta) = \Sigma, \quad (16)$$

$$U = -\kappa(\Phi)\Pi_\xi, \quad \kappa(\Phi) = (H(\Phi))^{-1}, \quad (17)$$

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