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Review of Discrete-Continuous Models in Energy and Transportation

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Abstract

Demand modelling of energy and transport incorporate interconnected decision variables, which are either discrete or continuous. After forty years, from McFadden multinomial logit model and incorporating it to Heckman endogenous simultaneous equation, there is an opportunity to determine the parameters of interconnected decisions simultaneously. These models are bounded by the utility theory. Now these models have matured, and their empirical aspects revealed. In this study, the pioneering works on discrete-continuous models that have been developed in the field of energy and transportation have been reviewed with a view of proposing new development to these models. These models theoretically are based on two approaches; McFadden indirect utility function and Gorman polar functional form of utility structures. In both approaches, the models are estimated by maximum likelihood procedures.

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1. Introduction

Most of the decisions in real world are interconnected and require to be taken sequentially or simultaneously. A sustain decision-making system will concern to the other agents demands. Sustainability considerations have forced the modeling system to search for multi-dimensional structures which have the ability to incorporate more than one dependent variable.

On consumer demand behavior, one may encounter a qualitative or discrete variable in addition to traditional continuous analysis of demand; and this is known as a discrete-continuous framework. For instance, a consumer decides on which particular brand of car to buy as well as how long to drive or use it; or a shipper decides on which mode of transport to be used while also concerns about the shipment size. In terms of housing, a consumer decides whether to buy or rent a house coincides with the size of house to live in; or a consumer decides whether to buy an electric or gas heater as well as the amount of electricity usage in a month. These examples require modelers to be concern with more than one subject at a time. These models are first developed in the field of energy as well as transport. In all cases the choice of discrete variable depends partly on the continuous variable and vice-versa. Therefore the two choices should be mutually and consistently modeled. Discrete-continuous model has been applied in energy economics and transportation fields.

In order to model discrete-continuous dependent variable systems two approaches have been proposed in the literature. The first is based on the indirect utility function which is proposed by Dubin and McFadden [1], Hanemann [2], Heckman [3]. The second one is the utility-based (Gorman structured utility function) and multiple discrete-continuous extreme value (MDCEV) model which is proposed by Bhat and Sen [4]. Currently, development in discrete model estimation provided an opportunity to simultaneously estimate a system of equations that include more than one discrete and continuous dependent variables. Identification of new developments in this type of models is the main goal of this study. For this review only pioneering works have been selected and their modeling procedure has been explained. Subsequently future model development works are proposed.

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2. General framework of discrete-continuous models: Indirect utility function and Gorman polar forms

Based on McFadden [5] a consumer indirect utility function gives the consumer maximal utility with respect to product price level and consumer income. It is a consumer preference based on market condition and income level. Utility maximization problem (UMP) based on price and income level is:

$$V(p,I) \equiv max_X \left\{ U(X) \middle| I - \sum_i p_i \cdot x_i \geq 0 \text{ and } X \geq 0 \right\}$$

where; V is observed part of utility function for product x; p is price and I is income.

From the above equation the maximum attainable level of utility as a function of price and income is presented. If there is a unique solution to the consumer's UMP, thereafter Marshallian demand function can simply be substituted into the utility function as:

$$V(p_1, p_2, \dots, p_n, I) \\ \equiv U\big(x_1^*(p_1, p_2, \dots, p_n, I), \dots, x_n^*(p_1, p_2, \dots, p_n, I)\big)$$

where; $x^*=x(p, I)$ and solves $maxU(x)$ subject to budget constraint $(p \cdot x \leq I)$. Here indirect utility value V is determined based on the value of U and any rescaling of U like density function of U is applicable and analogous to V. That is indirect because consumers usually care about what they consume rather than the prices, or "the demand for durables arises from the flow of services provided by durables ownership, the utility associated with a consumer durable is then best characterized as indirect" [1]. The indirect utility function unlike random utility function embodies an optimization process where consumers through a bundle of products try to maximize their utilities. This presumption let the consumers to choose a bundle which is affordable and maximizes their utilities. Definition of indirect utility function and how to optimize the decision will impose some properties which any indirect utility will posses. Some of these properties are:

1. Homogeneous of degree zero in price and income: If prices and income multiplies by the same positive factor, the budget constraint does not change, and therefore the choice and the utility level will not change; $V(t \cdot p, t \cdot I) = V(p, I)$ for $t>0$.
2. Non-increasing in prices and non-decreasing in income.
3. Quasi-convex in prices and income: If we draw the level curves of $v(p,m)$ in the space of (p_1,p_2) all maximum utility curves will be convex towards the origin.
4. Continuous in prices and income.
5. Roy's identity if $V(p, I)$ is differentiable [5]:

$$x_i^*(p_1, \dots, p_n, I) = -\frac{\partial V / \partial p_i}{\partial V / \partial I}, \quad i = 1, 2, \dots, n$$

Roy's identity is the main and useful property of indirect utility function in the case of demand analysis.

The foregoing notations are conditional to the chosen commodity. Here, the discrete choice of which commodity to be selected in a dual-product situation can be represented by a set of binary values $\Delta = (\delta_1, \delta_2, \dots, \delta_n)$ where; $\delta_j=1$ if $x_j>0$ and $\delta_j=0$ if $x_j=0$. The discrete choice can be represented in terms of the conditional indirect utility function as:

$$\delta_j(p, b, y, s, \varepsilon) \\ = \begin{cases} 1 \text{ if } v_j(p_j, b_j, y, s, \varepsilon) \geq v_i(p_i, b_i, y, s, \varepsilon) & \text{for all } i \\ 0 & \text{otherwise} \end{cases}$$

where; b is the attributes of x's commodity; y is income level; s is decision-maker characteristics like as age, education, …;ε is random component of utility with some joint density function.

Here, for an econometrician or observer, the discrete choice indices δ's are random variables with expected value of $E(\delta) \equiv \pi_j$ given by:

$$\pi_j(p, b, y, s, \varepsilon) \\ = Prob\{v_j(p_j, b_j, y, s, \varepsilon) \geq v_i(p_i, b_i, y, s, \varepsilon), for\ all\ i\} \\ = \int_{-\infty}^{+\infty} F_v^j(u, \dots, u)du,$$

where; F_v^j is the derivative of $F_v(.)$ with respect to its jth argument.

The unconditional function and conditional function is related to each other based on the value of δ which is 0 or 1(selected x and its correspondent indirect utility is showed with – mark). Then, this relation will be:

$$x_j(p, b, y, s, \varepsilon) = \delta_j(p, b, y, s, \varepsilon) \cdot \bar{x}_j(p_j, b_j, y, s, \varepsilon)$$

which $j = 1, \dots, N$

$$v(p, b, y, s, \varepsilon) \\ = max[\bar{v}_1(p_1, b_1, y, s, \varepsilon), \dots, \bar{v}_N(p_N, b_N, y, s, \varepsilon)]$$

Econometricians have used these relations to draw distributions of x_j and v. To do this, they introduced the sets $A_j \equiv [\varepsilon |\ v_j >= v_i]$. From f_ε one can construct $f_{\varepsilon|\varepsilon \in A_j}$ as a conditional joint density of $\varepsilon_1, \dots, \varepsilon_m$ given that $\varepsilon \in A_j$ then the probability density of x_j, $f_{\varepsilon|\varepsilon \in Aj}(x) = prob[x_j=x \mid \varepsilon \in A_j]$ is obtainable by analogy from $f_{\varepsilon|\varepsilon \in Aj}$ by a change of variable based on Roy's identification. Thus, the probability density of x_j, $f_{xj}(x)=prob(x_j=x)$ takes the following form:

$$f_{x_j}(x) = f(x) = \begin{cases} 1 - \pi_j, & x = 0 \\ \pi_j f_{x_j|\varepsilon \in A_j}(x), & x > 0 \end{cases}$$

Then, conditional indirect utility function and the joint density of random components are milestones of the random utility discrete-continuous models. If one specifies v_j and f_ε then, densities of f_v and $f_{\varepsilon|\varepsilon \in Aj}$ can be constructed which are used to form the discrete choice probabilities and the conditional and unconditional densities of x_j's [2].

A close consideration to the unconditional and conditional function for the assumed binary model will let to value of x in the following form based on Roy's identity to be expressed: