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Taking into account unbounded displacements in tolerancing analysis

Lazhar Homri*, Denis Teissandier, Alex Ballu

Univ. Bordeaux, I2M, UMR 5295, F-33400 Talence, France.

*Corresponding author. Tel.:+33-5 4000 8790; fax: +33 5 4000 6964. E-mail address : l.homri@i2m.u-bordeaux1.fr

Abstract

In tolerancing analysis area, a classical approach consists in handling sets of linear constraints. These sets of constraints characterize the boundaries of the relative displacements between two surfaces of the same workpiece or between two surfaces of two different parts, potentially in contact. The relative position between any two surfaces of a mechanism is determined by operations on these sets of constraints (Minkowski sum and intersection). A method for solving these operations is to model each set of constraints by a polytope, which by definition is a bounded intersection of many finitely closed half-spaces in some $\mathbb{R}^n$. However, the intersection of half-spaces simulating geometric constraints or contact is generally not bounded. This is due to the degree of invariance of a surface and the degree of freedom of a joint characterizing theoretically unbounded displacement. This article introduces the concept of "cap" half-spaces to delimit sets of constraints in $\mathbb{R}^6$. They are added to the operand set and in this way determining the relative position of two surfaces of a mechanical system is based solely on operations on operand polytopes generating a calculated polytope. By checking that a calculated polytope is included within a functional polytope the conformity of a mechanical system can be simulated with respect to a functional requirement. The addition of cap half-spaces to the operand sets will affect the topology of a calculated polytope. Hence it has to be possible to differentiate among all the facets of a calculated polytope between those that are generated by the cap half-spaces and the others generated by half-spaces that derive from geometric and contact constraints. This is essential in order to validate the geometric tolerances that ensure that a mechanical system is compliant in relation to a functional requirement. This article describes how to identify the facets generated by the cap half-spaces of a polytope resulting from a Minkowski sum or an intersection between two operand polytopes.

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Keywords: Tolerance analysis; polytope; cap half-spaces; Minkowski sum; degrees of freedom

1. Introduction

The aim of tolerancing analysis is to verify the mechanical system compliance with respect to functional requirements in terms of geometric specifications of constituent parts and contact specifications between parts potentially in contact.

A geometric specification is defined by a set of constraints which characterizes all the possible positions of a real surface within a tolerance zone. A tolerance zone is a region bounded by a perfect geometry, offset from the nominal surface [1]. Similarly, a contact specification is defined by a set of constraints that characterizes all the relative positions between two surfaces of two distinct parts potentially in contact [2].

In general, these sets of geometric constraints or contact constraints are operand sets that may be conformed to sets of half-spaces of $\mathbb{R}^6$. Giordano showed that the relative position of two parts resulting from several potential contacts can be formalized by an intersection operation on sets of contact constraints [3]. Fleming established the correlation between cumulative defect limits on parts in contact and the Minkowski sum of finite sets of geometric constraints [2]. More generally, the relative position between two surfaces of a mechanical system is characterized by a set of half-spaces, determined by operations (intersection and Minkowski sum) on operand sets [4,5]. Each operand set

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is a polyhedron of $\mathbb{R}^6$. A polyhedron is an intersection of a finite number of closed half-spaces of $\mathbb{R}^6$. We distinguish two types of polyhedron: a geometric polyhedron associated to geometric constraints and a contact polyhedron derived from contact constraints. In general, a polyhedron defined in geometric tolerancing is not bounded. Indeed, the displacements leaving globally invariant a surface and the displacements corresponding to the degrees of freedom of a joint are not limited by constraints. Minkowski sum of polyhedra can induce a prohibitive computational complexity. To overcome this problem, we have chosen to work only with polytopes. A polytope of $\mathbb{R}^6$ is a bounded polyhedron. Algorithms of Minkowski sums of polytopes dedicated to tolerance analysis where developed [6].

This article describes how to identify the facets generated by the cap half-spaces of a polytope resulting from a Minkowski sum or an intersection between two operand polytopes.

In the following, we limit ourselves to 6-dimension polyhedra and polytopes: the half-spaces arising from the geometric and contact constraints are linear inequalities in six variables: three rotation variables and three translation variables [4].

In the first part, some properties of polyhedra and polytopes are considered; the second part looks at determining the cap half-spaces which set boundaries to the half-space intersections resulting from geometric and contact constraints.

The third and the fourth parts deal with the two methods of identifying the dependent facets of cap half-spaces in a summation and an intersection respectively. Finally, we will discuss on the main advantages of this method and some future developments will be presented.

In this article, we put forward the following physical hypotheses:

- no form defect in the real surfaces,
- no local strain in surfaces in contact,
- no deformable parts.

2. Preliminaries on polytopes

2.1. Polyhedron, polytope, face

A polyhedron is an intersection of finitely many closed half-spaces in $\mathbb{R}^n$ (see Fig. 1(a)) [7,8]. This is the $\mathcal{H}$- description of a polyhedron [7]. We choose in this article, a set of m half-spaces $\bar{H}^- = \{\mathbf{x} \in \mathbb{R}^n : \ \mathbf{a}^T\mathbf{x} \le b\}$ to define a polyhedron $\mathcal{P}$ according to (1).

$$\mathcal{P} = \bigcap_{i=1}^{m} \bar{H}_i^- = \left\{\mathbf{x} \in \mathbb{R}^n : \ \mathbf{a}_i^T\mathbf{x} \le b_i \ , \ i = 1,...,m\right\} \\ = \left\{\mathbf{x} \in \mathbb{R}^n : \ \mathbf{Ax} \le \mathbf{b}\ \right\}, \mathbf{A} \in \mathbb{R}^{m\times n} \text{ and } \mathbf{b} \in \mathbb{R}^m \tag{1}$$

Where $\mathbf{a}_1{}^T,...,\mathbf{a}_m{}^T$ are the rows of $\mathbf{A}$ and $b_1,...,b_m$ are the components of $\mathbf{b}$.

A $\mathcal{H}$- polytope is a bounded $\mathcal{H}$- polyhedron (see Fig. 1(b)). A d- polytope is a polytope of dimension d in some $\mathbb{R}^n$ ($d \le n$). A 0-polytope is a vertex, a 1-polytope is an edge and a 2-polytope is a polygon.

A hyper-plane H is a support hyper-plane of $\mathcal{P}$ if:

$$\mathcal{P} \cap H \ne \varnothing \text{ and } \mathcal{P} \subset \bar{H}^- \tag{2}$$

A face F of $\mathcal{P}$ is the intersection between $\mathcal{P}$ and one of its support hyper-planes. The faces of a d-polytope $\mathcal{P}$ are convex sub-sets of dimension k, $0 \le k \le d-1$ [9, 10].

A face of dimension d is denoted d- face. A 0-face is a vertex, a 1-face is an edge and a ($d-1$)-face is a facet of $\mathcal{P}$.

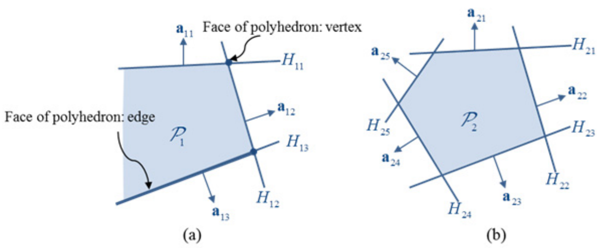

Fig. 1. (a) A 2-polyhedron $\mathcal{P}_1$; (b) a 2-polytope $\mathcal{P}_2$

2.2. Dual cone, normal fan

A cone is a non-empty set of vectors that, with any finite set of vectors, also contains all their linear combinations with nonnegative coefficients [7]. In particular, every cone contains the origin. For any arbitrary subset $Y = \{y_1,...,y_d\} \subseteq \mathbb{R}^n$, the cone associated to Y is defined as [7]:

$$\text{cone}(Y) = \{Y\mathbf{t}, \mathbf{t} \ge 0\} = \{t_1.y_1 + ... + t_d.y_d \ , t_i \ge 0\} \tag{3}$$

Some examples of cones of dimensions 2 and 3 are given in [6,7].

Every vertex v of a polytope $\mathcal{P}$ has an associated primal and dual cone as:

- The primal cone is composed of sets of edges and facets associated to the vertex v.
- The dual (or normal) cone is constructed as :