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A robust scheduling approach for a single machine to optimize a risk measure

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Abstract

Robustness in scheduling addresses the capability of devising schedules which are not sensitive – to a certain extent – to the disruptive effects of unexpected events. The paper presents a novel approach for protecting the quality of a schedule by taking into account the rare occurrence of very unfavourable events causing heavy losses. This calls for assessing the risk associated to the different scheduling decisions. In this paper we consider a stochastic scheduling problem with a set of jobs to be sequenced on a single machine. The release dates and processing times of the jobs are generally distributed independent random variables, while the due dates are deterministic. We present a branch-and-bound approach to minimize the Value-at-Risk of the distribution of the maximum lateness and demonstrate the viability of the approach through a series of computational experiments.

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1. Introduction and Problem Statement

In real production environments, scheduling approaches have to deal with the occurrence of unexpected events that may stem from a wide range of sources, both internal and external. Production activities may require more time or resources than originally estimated, resources may undergo failures, materials may be unavailable at the scheduled time, release and due dates may change and new activities like rush orders or reworks could be inserted in the schedule. Robust scheduling approaches aim at protecting the performance of the schedule by anticipating to a certain degree the occurrence of uncertain events and, thus, avoiding or mitigating the costs due to missed due dates and deadlines, resource idleness, higher work-in-process inventory.

The vast majority of the stochastic scheduling literature considers the stochastic aspect of a problem in terms of a scalar performance indicator, e.g., the expected value. When addressing a scheduling problem, the capability of minimizing the expected value of an objective function provides a significant improvement respect to pure deterministic approaches. However, the expected value is not suitable to exhaustively model the quality of the schedule from the stochastic point of view [1,2].

As an example, minimizing the expected value of the maximum lateness aims at assuring an average good performance in terms of due date meeting but does not protect against the worst cases if their probability is low. Protection against worst cases is a natural tendency in management decisions. Plant managers who face uncertainty try to maximize the mean profit but also try to avoid the rare occurrence of very unfavourable situations causing heavy losses. To cope with this problem, the financial literature has proposed risk measures able to consider the impact of uncertain events both in terms of their effect and of their occurrence probability [3,4]. In the scheduling area, on the contrary, risk analysis and assessment are not so popular even if the concept of risk is often perfectly suitable to support scheduling decisions under uncertainty. Against its potential utility, the application of risk measures to scheduling problems has not be extensively addressed due to the difficulty in considering the objective function in terms of its stochastic distribution instead of a scalar performance indicator (i.e. expected value, variance) [5].

In this paper we consider a stochastic scheduling problem with a set of n jobs that must be sequenced on a single machine. This can model a single machine as well as a group

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of resources, or a whole department. Although it could seem a restrictive hypothesis, a single resource model is applicable to several cases where a group of resources can only work on a single product or a single product type at a time (e.g., multi-model transfer lines, make-to-order shops working on a single job or batch at a time). The aim is at optimizing a risk measure of the maximum lateness using a branch-and-bound algorithm. The processing times p_j of the jobs are generally distributed independent random variables. The jobs are available after a release date r_j and have a due date d_j. The release dates r_j are also generally distributed independent stochastic variables while the due dates d_j are deterministic. The objective of the scheduling problem is to optimize a stochastic function of a given performance measure. In particular we focus on the *maximum lateness* $L_{max} = max\{L_j, j = 1, \cdots, n\}$, with $L_j = C_j - d_j, j = 1, \cdots, n$ where C_j is the completion time of job j under the given schedule. This objective function is likely to minimize a stochastic function of the maximum magnitude of the deviations with respect to the due dates, thus protecting the schedule from the impact of the worst cases.

In Section 2 the present advances for the existing stochastic scheduling approach are summarized. Section 3 reports an outline of the risk measure used, the *Value-at-Risk* (*VaR*). Section 4 describes the principles and operation of the proposed branch-and-bound solution method.. Section 5 reports on the computational test result, while Section 6 concludes the paper.

2. State of the Art

The deterministic version of the considered stochastic problem is known as $1|r_i|L_{max}$ and has been recognized to be strongly *NP*-hard [6]. A review of the existing solution approach for this scheduling problem can be found in [7] and [8, chap.9]. If we do not consider the release times, the resulting scheduling problem ($1|L_{max}$) is rather simple and can be solved to optimality using the *earliest due date* (EDD) rule.

Referring to the stochastic counterpart, when considering a single machine scheduling problem with arbitrarily distributed processing times and deterministic due dates, the EDD rule still minimizes the expected maximum lateness [8]. This applies to non-preemptive static list and dynamic policies, as well as to preemptive dynamic policies. These results ground on the fact that the EDD rule minimizes the maximum lateness of the deterministic version of the problem. Hence, given any realization of the processing times, the EDD rule provides the optimal schedule and, since this happens for all the realizations, then the EDD rule minimizes the maximum lateness also in expectation [8].

This result has further implications on the maximum lateness distribution. Since the EDD schedule provides the optimal maximum lateness for any realization of the processing times, given a maximum lateness L^* and a schedule S^*, the probability of having $L_{max} \leq L^*$ must be less or equal to the value obtained with the EDD schedule. Due to this, the cumulative distribution of the maximum lateness for the EDD schedule bounds from above all the cumulative distributions of the maximum lateness for any possible schedule. This behavior can be formalized in terms of stochastic order relations [9,10, chap.9].

The relationships between rearrangement inequalities and scheduling problems have been addressed in [11]. Using stochastic rearrangement inequalities, the author obtains a solution for the stochastic counterpart of many classical deterministic scheduling problems. These results have been rephrased and further exploited in [12–15].

It must be noticed that part of the stochastic scheduling literature addresses the problem of minimizing the maximum expected lateness $max(E[L])$. In this problem, using a stochastic function $E[L]$, the stochastic problem is reduced to a deterministic minimization [12]. On the contrary, considering the minimization of the expected value of the maximum lateness $E[L_{max}]$ retains the stochastic characteristics of the scheduling problem by regarding the whole distribution of the objective function.

A stochastic problem belonging to this class is analyzed in [15] where a set of jobs with deterministic process times and stochastic due dates are scheduled on a single machine to minimize the expected value of the maximum lateness ($E[L_{max}]$). The authors propose a dynamic programming algorithm and compare its performance to three different heuristic rules. The dynamic programming algorithm is also extended to cope with stochastic processing times and due dates. However, the provided results ground on the assumption that both the processing times and due dates are exponentially distributed.

Analogously to the deterministic case, when the release times are considered (either deterministic or stochastic), the problem becomes more difficult to solve. However, considering independent generally distributed release times and independent generally distributed processing times, if the due dates are deterministic, the EDD rule still minimizes L_{max} but only in the preemptive case [8]. Some further extensions are available but only assuming that the due dates are deterministic but both the release and processing times are exponentially distributed with the same mean [8].

Referring to the use of stochastic objective function other than the expected value, the most common is the variance. In fact, a trade-off between mean and variance is one of the most simple and common risk measure. A joint optimization of expectation and variance in a single machine scheduling problem has been proposed in [16]. Other common objective functions in the stochastic scheduling are the flow time and the completion time. Moreover, in a recent paper [17] provides closed form equations of mean and variance for a large set of scheduling problems. However, no algorithm, neither exact, nor heuristic, has been proposed for the maximum lateness single machine scheduling problem to optimize a stochastic objective function different from the expected value.

3. Risk Measures

Financial research has paid particular attention to the definition of risk measures to cope with uncertainty. In particular the study of extreme events, i.e., the tails of the distribution has received due attention. Risk measures as the *Value-at-Risk* are extensively used in portfolio management and a large amount of literature have been written on their mathematical properties and effectiveness in protecting assets investments.

According to the notation used in [18], we consider a vector of decision variables x and a random vector y governed by a probability measure P on Y that is independent on x. The decision variables and random vectors x and y univocally