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Max-Plus Modeling of Manufacturing Flow Lines

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Abstract

Max-plus algebra can be used to model manufacturing flow lines using linear state-space-like equations which can be used in analysis and control. This paper presents a method for easy and quick generation of the max-plus equations for manufacturing flow lines of any size or structure. The generated equations can model flow lines with infinite as well as finite buffer sizes.

A flow line to be modeled is initially assumed to have infinite buffers for all stations. The line model equations are then generated as a combination of serial and merging stations after identifying the different stages using an adjacency matrix for the flow line. In the generated equations, the dynamics of the system are captured in two matrices that are function of the processing times of the different stations in the line. After generating these equations, extra terms are added to account for the finite buffers where for each buffer size, a matrix is added multiplied by the vector of system parameters delayed by the buffer size plus one.

The method is intuitive and easy to understand and code in software and thus can facilitate quick analysis of different configurations of manufacturing flow lines and assessing what if scenarios. This can also allow quick on-line reconfiguration of controllers for frequently reconfiguring flow lines.

Keywords: *Flow lines; Max-plus alegbra; Parametric modeling*

1. Introduction

Manufacturing systems fall under the category of Discrete Event Dynamic Systems (DEDS). For these systems, modelling tools include automata, petri-nets, markov-chains, queuing networks, simulation, and max-plus algebra [1]. Among these models, max-plus algebra is the only tool that can model the system using linear algebraic equations analogous to conventional state-space linear equations [2]. Using these equations, real time control as well as parametric system analysis becomes possible.

The use of Max-plus in modelling discrete event systems is fairly new starting in 1984 and since then it has been used in many applications in manufacturing systems including: manufacturing systems modelling [3, 4], performance evaluation [5, 6], performance optimization [7], and control [8, 9].

A corner stone in modeling, performance evaluation, performance optimization or control of manufacturing systems using max-plus algebra is obtaining the equations that describe the system. For relatively small and simple systems, these equations can be derived easily by hand, however; for large and complex systems, obtaining these equations is difficult, tedious and time consuming. In this paper, a method for easy and quick generation of the max-plus system equations for flow lines is presented. Flow lines studied in this paper are assumed to have deterministic processing times and reliable machines. The first assumption is realistic for automated systems as well as semi-automated systems with palletized material handling where the process time variation is much less than the processing time and thus can be neglected. The second

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assumption is also realistic when studying the normal short-term system operation with the objective of understanding and optimizing the system behavior as opposed to studying long-term operation with the objective of planning system capacity where machines breakdown would have an effect.

A review of related research is presented in section 2. Section 3 presents a review of the basics of max-plus algebra; section 4 presents the method for generating the max-plus equations and section 5 presents the discussion and conclusions.

2. Related research

Modeling simple manufacturing systems using max-plus algebraic equations is easy and intuitive. The necessary conditions for each station to start operating on a job can be extracted from the description of the system and then written as a combination of addition and maximization operations. These equations can then be put together in a state-space matrix form, where the system parameters are the starting (or finishing) times of operating of the stations, and the system matrices are formed of the processing times of these stations. However, as the systems grow in size and/or have a complicated structure, generating the model equations becomes less intuitive, tedious and time consuming. In addition, modeling finite buffers with max-plus equations is not straight-forward or easy. Several papers have been published focusing on facilitating the modeling of manufacturing systems using max-plus algebra. Doustmohammadi and Kamen [10] presented a procedure for direct generation of event-time max-plus equations for generalized flow shop manufacturing systems. The procedure generates the equations directly only for serial flow lines with one station in each stage. In more complicated cases, the equations are generated for each machine separately, interconnection matrices which describe the flow of jobs through the line are derived and then the final equations are generated using matrix manipulations and several recursions. In addition, the procedure is limited to flow shops with infinite buffers. In [11], Goto et al. proposed a representation form for manufacturing systems that can account for finite buffers by adding relations between future starting times of jobs on a station and the past starting times for the same station and the following one. Imaev and Judd [12] used block diagrams which can be interconnected to form a manufacturing system model. This approach also assumes infinite buffer sizes.

In summary, the literature is lacking a tool that can easily generate max-plus equations for flow lines that are complex and contain finite buffers.

3. Basics of max-plus algebra

Max-plus algebra is an algebraic structure in which the two allowable operations are “maximization” and “addition”. In this section an introduction to the basic concepts and tools of the max-plus algebra will be presented.

Max-plus algebra is defined over $\mathcal{R}_{max} \rightarrow \{ \mathcal{R} \cup -\infty \}$ where $\mathcal{R}$ is the set of real numbers. The two main algebraic operations are maximization, denoted by the symbol $\oplus$, and addition, denoted by the symbol $\otimes$ where:

$$\mathrm{a} \oplus \mathrm{b} = \max\ (\mathrm{a}, \mathrm{b}) \quad \forall\, \mathrm{a}, \mathrm{b}\ \in \mathcal{R}_{\max}$$
$$\mathrm{a} \otimes \mathrm{b} = \mathrm{a} + \mathrm{b} \quad \forall\, \mathrm{a}, \mathrm{b}\ \in \mathcal{R}_{\max}$$

The null element of the operation $\oplus$ is ε which is equal to $-\infty$, and the null element for the operation $\otimes$ is e which is equal to 0. This can by demonstrated by:

$$\mathrm{a} \oplus \varepsilon = \max\ (\mathrm{a}, -\infty) = \mathrm{a} \quad \forall\, \mathrm{a}\ \in \mathcal{R}_{\max}$$
$$a \otimes e = a + 0 = a \quad \forall\, a\ \in \mathcal{R}_{max}$$

Similar to traditional algebra, both $\oplus$ and $\otimes$ are associative and commutative, and multiplication is left and right distributive over addition:

$$a \otimes (b \oplus c) = (a \otimes b) \oplus (a \otimes c)\ \forall\, a, b, c\ \in \mathcal{R}_{max}$$
$$(a \oplus b) \otimes c = (a \otimes c) \oplus (b \otimes c)\ \forall\, a, b, c\ \in \mathcal{R}_{max}$$

Max-Plus algebra can be extended over matrices similar to conventional algebra. If $\boldsymbol{A}$ and $\boldsymbol{B}$ are two matrices with equal dimension then:

$$\boldsymbol{A} \oplus \boldsymbol{B} = \boldsymbol{C},$$

where $\boldsymbol{C}_{ij} = \boldsymbol{A}_{ij} \oplus \boldsymbol{B}_{ij}$. If the number of columns of $\boldsymbol{A}$ is equal to the number of rows of $\boldsymbol{B}$ equal to n, then:

$$\boldsymbol{A} \otimes \boldsymbol{B} = \boldsymbol{C},$$

where

$$\boldsymbol{C}_{ij} = \oplus_{k=1}^{n} (\boldsymbol{A}_{ik} \otimes \boldsymbol{B}_{ki}),$$

where $\oplus_{k=1}^{n} \boldsymbol{C}$ is maximization of all the elements of $\boldsymbol{C}$ over $k = 1$ to n.

Through the rest of the paper, the $\otimes$ operator will be omitted whenever its use is obvious, thus $a \otimes b \oplus c \otimes d$ will be written as $ab \oplus cd$.

An equation is the general form:

$$\boldsymbol{X} = \boldsymbol{A}\,\boldsymbol{X} \oplus \boldsymbol{B}\,\boldsymbol{U} \tag{1}$$

where $\boldsymbol{X}$ is an $n \times 1$ vector of variables, $\boldsymbol{U}$ is an $m \times 1$ vector of inputs, $\boldsymbol{A}$ is an $n \times n$ square matrix and $\boldsymbol{B}$ is an $n \times m$ matrix, has a solution [14]:

$$\boldsymbol{X} = \boldsymbol{A}^{*}\,\boldsymbol{B}\,\boldsymbol{U} \tag{2}$$

where $\boldsymbol{A}^{*}$ is defined as:

$$\boldsymbol{A}^{*} = e \oplus \boldsymbol{A} \oplus \boldsymbol{A}^{2} \oplus \dots \oplus \boldsymbol{A}^{\infty}.$$

A complete detailed description and analysis of the max-plus algebra can be found in [14] and [15].

4. Flow lines modeling

Modelling will start with a flow line with *n* serial stations, then *n* different lines merging (assembling) in one line, then a general flow line with multiple serial lines with multiple merging. Modelling stations with finite buffers will then be presented afterwards in section *4.4*.

4.1. Modeling ‘n’ serial stations

The most common structure of a flow line is a serial structure with *n* processing stations, one input of raw material $\boldsymbol{U}$, and one output of finished products Y as shown in fig. 1. Let U_k, Y_k, and $X_{i,k}$ be the time at which the raw material is made available to the line, the time at which the