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3-D elasticity buckling solution for simply supported thick rectangular plates using displacement potential functions

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ABSTRACT

This paper concentrates on the elastic buckling of thick plates. The shear strains in thick structures such as thick plates are important and thus, cannot be neglected. On the other hand, shear strains in the analysis of thick plates leads to complicity of governing equations. In this paper, with the use of displacement potential functions, the governing equations are simplified to two partial differential equations for the potential functions, one of which is second order and the other is forth order. These PDEs are established for a rectangular isotropic body, so that they are applicable to any arbitrary thickness of plate with no limitation on its thickness ratio. By solving the governing differential equations using separation of variables method and satisfying the exact boundary conditions, an analytical solution is obtained for linear elastic buckling of simply supported rectangular thick plates, subjected to in-plane either uniaxial or biaxial static loads, one of which could also be tensile force. Then critical buckling load is expressed in terms of non-dimensional buckling factor. The results of this paper are compared with other analytical and numerical works for thin and moderately thick plates, and also with numerical works for thick plates, which proves an excellent agreement between the results of this paper and others.

1. Introduction

It is well known that neglecting the transverse shear deformation will lead to an overestimate for the buckling load in comparison with more accurate methods. Reissner [1] proposed a simple plate theory, which takes the shear deformation into account for bending analysis. This theory was afterwards extended to vibrating plates including the rotary inertia by Mindlin [2], plate buckling by Herrmann and Armenakas [3] and also by Brunelle and Robertson [4]. In these plate theories, the stress distribution through the plate thickness is simply assumed to be constant. Thus, these theories use a shear correction factor to adjust the shear stress distribution in order to compensate the error.

Srinivas and Rao [5] developed a 3-D linear, small deformation solution for the bending, vibration and buckling of simply supported thick orthotropic rectangular plates and laminates in theory of elasticity point of view. They assumed that the displacements u, v and w to be produced by trigonometric functions of x and y multiplied by an unknown function of z. Although these kind of assumptions, which are made in most of current methods, make the solution easier, but they decrease the accuracy of the method and limit its applicability. Matsunaga [6] just treated the out of plane buckling problems of plates subjected to in-plane stress, however, he considered the in-plane and out-of-plane buckling of thick plates subjected

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to in-plane load in [7, 8] in which he clarified the applicability and reliability of the 2-D higher order theory through static boundary-value problems for an extremely thick plate.

The exact solutions for plate's problems are difficult to be achieved. Because of this, some numerical methods like finite stripe, finite element, and other methods are so common in this field. Dawe and Roufaeil [9] studied buckling of rectangular Mindlin plates employing two related methods; Rayleigh–Ritz method and its piece-wise form, the finite strip method. The applied membrane load can involve biaxial normal stress plus shear stress. By using the finite strip method, Plank and Wittrick [10] have offered a simple relation to relate the vibration and buckling problems. Benson and Hinton [11] and also Hinton [12] have solved the buckling of moderately thick plates with two simply supported opposite edges using the finite strip method considering the curvature terms; but their method is limited to the cases in which the strips are simply supported and have normal stresses at two edges. By employing the spline strip method, Mizusawa [13] analyzed the buckling problem of Mindlin plates with the thickness tapering linearly in one direction.

Rao et al. [14] studied the stability of moderately thick rectangular plates by using a high precision triangular finite element. By using a modified complementary energy principle, Luo [15] made the finite element analysis for the buckling of both thin and moderately thick plates. Teo and Liew [16] made the first attempt to use the Differential Quadrature (DQ) method to find the eigenvalue solution for 3-D buckling analysis of rectangular plate with general boundary conditions. Based on the Mindlin plate theory, Liu [17] applied the Differential Quadrature Element Method (DQEM) for buckling analysis of discontinuous rectangular plates. Civalek [18] applied the method of Discrete Singular Convolution (DSC) to find a numerical solution for 3-D analysis of thick rectangular plates and studied their free vibration, bending and buckling.

Several efforts have been devoted to buckling analysis of the plates with different boundary conditions, shapes or geometries, and likewise, for plates with inconstant thickness or under inconstant loadings. By applying numerical integration, Sakiyama and Matsuda [19] obtained an approximate solution for the differential equations of buckling for Mindlin plates in discrete forms. Since the method requires no initial assumptions for the plate deflection, it can be used for complex plates having mixed boundaries with good accuracy. By implementing the pb-2 Rayleigh–Ritz method on the basis of energy functional resulting by the incremental total potential energy approach, Wang et al. [20] focused on rectangular Mindlin plates under normal uniform in-plane load with internal line (straight or curved) supports and presented the buckling factor for several loading and boundary conditions. Liew and Liu [21] employed the differential cubature method to solve the arbitrarily shaped thick plates, for the first time. This method transforms the governing differential equations and boundary conditions into sets of linear algebraic equations.

A solution for buckling and vibration of stepped rectangular plates with a pair of opposite edges simply supported, based on the Mindlin first order theory, was presented by Xiang and Wei [22]. By employing the Galerkin–Bubnov theorem of identity, Radosavljević and Dražić [23] studied the buckling problem of rectangular CFCF (C: Clamped and F: Free edges) plates made up stepped thickness.

Liew et al. [24] applied the concept of state space to the Levy-type solution method and performed the first buckling analysis for thick plates having loaded free edges. Leissaa and Kang [25] found a solution for the free vibration and buckling of rectangular SCSC plates (S: Simply supported), with the simply supported edges subjected to a linearly varying normal load. They assumed that the transverse displacement (w) to be varied as a sine function. Since, the method of Frobenius has been used, the solution was obtained as a power series. Thai et al. [26] used two variables refined plate theory considering parabolic variation of transverse shear stress through the thickness of the plate without shear correction factor.

Liew et al. [27] employed the mesh-free method for the free vibration and buckling analyses for moderately thick plates by applying the First-order Shear Deformation Theory (FSDT). Also, in another paper, by using FSDT, Liew et al. [28] formulated the Radial Point Interpolation Method (RPIM) to analyze the buckling problem of plates subjected to non-uniformly in-plane loads. Moreover, for elastic buckling of plates under partially edge load, Liew and Chen [29] proposed a numerical algorithm based on RPIM.

Analysis of biaxially loaded plates, give some new results about buckling that makes these problems so different from the problems of uniaxially loaded plates. Wittrick [30] obtained the exact 3-D solution for eigenvalue problem for buckling of rectangular plates subjected to biaxial compression. Bui et al. [31] introduced a new mesh-free method to study the buckling of plates subjected to uniaxial or biaxial in-plane compression and pure shear stress. They stated that shear-locking phenomenon is firmly involved when thick plate theories are used to analyze thin plates. As well, they recognized that the shear-locking are due to the incompatibility between the rotation and the lateral displacement fields.

In recent years the mesh-free or element-free method is developed for solving of different plate problems. This method is used in several articles such as [32–37] by Zhang et al. and [38] by Zhu et al. for Functionally Graded Material (FGM) plates. By using the element-free Galerkin method, Jaberzadeh et al. [39] analyzed inelastic buckling of skew and rhombic tapered plates. Most of these works use governing equations based on simplifying assumptions and therefor are not classified as exact methods.

In cases which the in-plane loadings cause relatively large in-plane deformations in the plates, the effect of pre-buckling deformations may become more pronounced. Ziegler [40] showed that if the tensile load is large enough, the effect of in-plane pre-buckling deformation can affect the same as the shear deformation. Naderi and Saidi [41] presented a solution for buckling problem of thick rectangular orthotropic plates under uniaxial and biaxial loadings based on the finite displacement theory, and studied the effect of pre-buckling deformations on the critical buckling load.

One of the effective and efficient methods that can be used to solve elasticity problems is displacement potential functions. By using this method, usually, the system of differential equations is uncoupled or at least simplified through