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Equilibrium pricing strategies in retrial queueing systems with complementary services☆

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ABSTRACT

An unobservable retrial queueing system with complementary services is studied, in which customers are risk neutral and they have no information of the system upon arrival, but have to make the decision to join the system or not to maximize their expected benefits. In our model, one server provides an instantaneous service while the other offers a service with delayed customers, which is modeled as an M/M/1 retrial queue. The two services are complementary and the customer has no benefit from obtaining just one of them. Under various pricing schemes, we investigate the customers' reward-cost situations and the servers' price strategies in equilibrium. In each pricing scheme three scenarios are studied according to the different ownership of the two servers: (1) they are owned and operated by two different private agents; (2) they are owned and operated by a common private agent; or (3) one is owned and operated by a private agent, and the other by a public agent. Some numerical experiments of the equilibrium solutions are presented.

1. Introduction

In the last decades, the game-theoretic analysis of queueing systems with decentralized behavior of customers have been paid considerable attention. To reflect customers' desires for service and unwillingness to wait, some reward-cost structures are imposed to the systems. Arriving customers are allowed to make decisions to decide whether to join the system or not, based on different levels of information of the system on hand, to maximize their utility. An excellent and complete survey on this topic has been presented by Hassin and Haviv [1].

However, few works have been carried out for game-theoretic analysis of retrial queueing systems, although customer retrials are rather common phenomena in daily-life, industrial engineering, communication systems and business managements. In a retrial queueing system, blocked customers who find the server unavailable upon arrival may join a retrial pool called orbit to try again for service sometime later. It is evident that customer retrials influence the quality of service in many practical scenarios, and retrial queueing systems have been widely used to model telephone switching systems, computer and communication networks, call centers, etc. For example, Artalejo and Phung-Duc [2] discussed an M/G/1-type retrial queue with two-way communication, and Shin and Choo [3] dealt with an M/M/s queue with balking, reneging and retrials, both of which can be used in call centers. For a comprehensive discussion of the main models and methodologies on this topic, see [4,5].

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In principle, game-theoretic analysis of retrial queues becomes challenging due to the complicated or unavailable expressions of waiting time distribution and other measures. Kulkarni [6,7] studied a single server retrial queueing model with two types of customers under noncooperative and cooperative strategies. Elcan [8] considered a single server Markovian retrial queue under the classical retrial policy, and derived the socially optimal and equilibrium retrial rates. Hassin and Haviv [9] extended the model to the general service time case. Zhang et al. [10] dealt with a single server retrial queue, in which two types of customers were considered and the server was subject to breakdowns and vacations. All the papers mentioned above assumed that balking is forbidden. While balking is allowed, Economou and Kanta [11] investigated the retrial queueing systems with a constant retrial rate, where the equilibrium strategies of the customers and the social and profit maximization problems were discussed. Recently, Wang et al. [12] investigated the joining strategies of the customers in M/M/1 retrial queues. They derived the corresponding Nash equilibrium and social optimization balking strategies for all customers. A monopoly pricing issue in a retrial queue with delayed vacations for local area network applications was studied by Wang et al. [13].

It is observed that the work mentioned above assumed one server. However, in many practical occasions the customer may encounter complementary services. Four representative examples are as follows:

Example-1: Cloud computing system. In the cloud computing system, the scheduler can allocate a computing node for a request which is generated by a user. Each computing node can be modeled as a retrial queueing system to process the requests, and the user will pay the charges to both the scheduler and the computing node. Here, the services provided by the scheduler and the computing node are complementary.

Example-2: Cognitive radio system. In a cognitive radio system, secondary users (SUs) have access to unoccupied bands after they register for the service and then try their luck to catch a dedicated band. The service process can be modeled as a retrial queueing system and the SUs are also charged by the owner of the band. The server who is in charge of registrations and the owner of the band provide complementary services.

Example-3: Stochastic-demand inventory system. Nowadays, many firms choose to operate on a make-to-order mode for the high holding cost products. Thus, for a customer who comes to purchase an electronic product composed of many parts, he can immediately get some parts on hand, but cannot obtain the remaining parts of high holding cost. Thus, the customer may place an order to the seller and check after some random time to fulfill the purchase. This process to obtain high holding cost parts can be modeled as a retrial queueing system. Only when customers obtain all necessary parts we can say the purchase is successful and completed.

Example-4: Cable TV networks. Complementary services are common in the occasion where cable TV networks are ungraded to realize bidirectional communications between network terminations (NTs) and a centrally located head end (HE). A mechanism composed of two stages can be set to make transmissions from NTs to HE coordinated. At the first stage, an NT having a data to transmit informs the HE of the number of data slots it needs by sending a request in a dedicated time slot. This time slot can only deal with one request at a time. If there are no other requests simultaneously in this time slot, the request can be transmitted successfully, otherwise the requests will be stored in the NT which will try to send the request again sometime later. This process can be modeled as a retrial queueing system. If a request is transmitted successfully, the HE starts the second stage. At this stage the HE sends a signal to the corresponding NT to permit it to transmit its data in specified data slots. In this situation, the services provided by the time slot transmitting the request and the data slots transmitting data are complementary, and the data that needs to be transmitted must receive both services (for details, see [14]). The owners of the time slot transmitting the request and the data slots transmitting data can charge the NT respectively, and the services provided by them are complementary.

In the literature, Veltman and Hassin [15] first studied a single-arrival queueing system with two complementary services, where the queueing model was assumed to be an M/M/1 queue. Under two pricing structures, they investigated the equilibrium solutions and compared them with the socially optimal results. Later, Sun et al. [16] extended the analysis to the batch-arrival queueing system and Ma et al. [17] to the multiple vacations queueing system. To the best of our knowledge, there is no work concerned with the equilibrium analysis of retrial queues with complementary services.

This paper aims to study the equilibrium solutions of the retrial queueing system with two complementary services. In our model, arriving customers will be served instantaneously by one of the servers and then join a "queue" to wait to be served by the other server. Throughout the paper, following the example of cloud computing system, we will call two servers "scheduler" and "service provider", respectively. We analyze our model under three pricing structures. In the first case, one of the two servers charges a fixed price, while the other one asks for a price that is proportional to the customer's sojourn time in the system. It is called "mixed pricing scheme". In the second case, both servers charge fixed prices regardless of the sojourn time. In the last case, two servers charge the service fees based on the sojourn time. We call them "fixed pricing scheme" and "time-based pricing scheme" respectively.

To summarize, our contributions are threefold. First, we investigate the retrial queueing system with complementary services for the first time. Second, using the Stackelberg game approach, we derive the optimal arrival rate of customers and the prices set by the servers in equilibrium. Third, three price structures and three scenarios of ownership are investigated extensively.

The rest of this paper is organized as follows. Section 2 presents the description of the basic model. In Section 3, the mixed price structure is discussed under three scenarios of ownership. The customers' reward-cost situations and price strategies of the servers in equilibrium are investigated. Sections 4 and 5 give the analysis of the fixed and time-based pricing schemes, respectively. Finally, numerical examples and conclusions are given in Sections 6 and 7, respectively.