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Stability and Hopf-bifurcation in a general Gauss type two-prey and one-predator system

B.D. Deka[a], Atasi Patra[b], Jai Tushar[b], B. Dubey[b,*]

[a] Department of Mathematics, Darrang College, Tezpur-784001, India
[b] Department of Mathematics, Birla Institute of Technology and Science, Pilani-333031, India

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ABSTRACT

A Gauss type general prey–predator mathematical model is proposed and analysed to study the effect of predation on two competing prey species. The growth rate and functional responses are taken to be general non-linear functions. By analysing the model, local stability of all possible equilibrium points is discussed. By choosing a suitable Lyapunov function the global stability of the system at positive equilibrium point is also found. For the purpose of numerical simulation, growth rates of both prey species are taken to be logistic and the predator's functional response on the prey species are taken as Holling type-II. Taking death rate of the predator as a bifurcation parameter, we observe Hopf-bifurcation of the system. Then we have discussed the stability and direction of the Hopf-bifurcation. We also observed that intra-specific interference factor is an important parameter in governing the dynamics of the system.

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1. Introduction

Mathematical models in ecology discuss about the dynamical behaviour of the ecological system. In ecology, the dynamics of biological species depends on many factors, such as intra and inter-specific interference, death rate, stage structure and environmental conditions, etc. In mathematical ecology, many researchers have conducted their research on prey–predator relations. The interaction of two prey and one predator which has the switching property of predation has been discussed in [1]. Kazarinoffa and Driessche [2] analysed a predator-prey model with a general functional response and competition among prey. They also discussed the criteria for the stability of small amplitude periodic solutions of the system.

Harrison [3] proposed and analysed the global stability of predator–prey interaction using Lyapunov's function. He also showed that conclusions do not depend so much on the specific function chosen by the modeler, but more on their general properties. Cheng et al. [4] and Liou and Cheng [5] discussed model for predator–prey interactions with different functional response. The qualitative effects of constant rate stocking of either or both species in a predator–prey system has been discussed in [6]. Hsu [7] analysed the effect of predation on the two competing prey species. He observed that the outcomes depend critically on the prey species capability of invading the complementary sub community formed by predator species and other prey. He also proved that competing prey species can coexist even with exactly identical resource requirements if each prey species has invasion potential for the complementary predator–prey sub community.

* Corresponding author. Tel.: +911596-515632; fax: +911596244183.
E-mail addresses: barnalidasdeka@gmail.com (B.D. Deka), atasimaths@gmail.com (A. Patra), jaitushar93@gmail.com (J. Tushar), bdubey@pilani.bits-pilani.ac.in, balram.dubey@gmail.com (B. Dubey).

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In recent years, some mathematical models have been proposed and analysed taking into account the relationship between prey–predator. Dubey [8] described a mathematical model to study the effects of two interacting population on the depletion of resource. In this model, it has been assumed that the resource is a common food for both the populations and one population is a supplementary food for the other. Dubey and Upadhyay [9] proposed and analysed a model with one prey and two predator with ratio-dependent predators growth rate. They also discussed the permanent co-existence of the three species. Rai and Upadhyay [10] studied the dynamics of a food chain model which depends on the behaviour of the top-predator. They considered a food chain model with specialist and generalist top-predators. Food chains with specialist top-predators are dictated by exogenous stochastic factors. They also discussed chaos and short-term recurrent chaos in the system. Rai et al. [11] described the dynamical complexity of five non-linear deterministic predator–prey model system. In this model these simple systems were selected to represent a diversity of tropic structures and ecological interactions in the real world. They found that these systems can dramatically change attractor types, and the switching among different attractors is dependent on system parameters. They also discussed chaotic behaviour of the system. Upadhyay and Naji [12] discussed analytical and numerical behaviour of a three species food chain model, consisting of a hybrid type of prey-dependent and predator-dependent functional responses. They also discussed the persistence of the system. The results show that the system exhibits rich complexity features such as stable, periodic and chaotic dynamics. Eletteby [13] proposed a multi-team prey–predator model, in which the prey teams help each other. He studied the local stability behaviour of the system. In the absence of predator, he proved that there is no help between the prey teams. Then he discussed the global stability and persistence of the model without help. A predator–prey model with Holling Type-II functional response incorporating a constant prey refuge has been analysed in [14]. They investigated instability and global stability of the equilibria, existence and uniqueness of limit cycle. Zhang and Huo [15] investigated a non-autonomous ratio-dependent predator-prey system with multiple exploited terms. Using coincidence degree theory, they established the existence of at least four positive periodic solutions. Upadhyay et al. [16] discussed a model with one prey and two predator with Holling type-II and Crawly-Martin type functional responses. They proved that periodic doubling reversals can generate shot-term recurrent chaos. In numerical simulation part, they suggested that periodic doubling reversals could control chaotic dynamics in ecological models. Lan and Zhu [17] discussed Hopf-bifurcation and limit cycle in the Holling–Tanner predator prey model. Upadhyay and Raw [18] described a dynamical behaviour of a three species food chain model with Holling type-IV predator response. They discussed the persistence criterion of the food chain model. The numerical bifurcation analysis reveals the chaotic behaviour in a narrow region of the bifurcation parameter space for biologically realistic parameter values of the model system. Braza [19] described a predator–prey model with a modified Lotka–Volterra interaction term as a functional response of the predator to the prey. The interaction term is proportional to the square root of the prey population. Mukhopadhyay and Bhattacharyya [20] discussed the vole population dynamics under the influence of specialist and generalist predation. They also assumed that the vole population and specialist predator grow logistically and the general predator depends for food on vole population with Holling type-III functional response.

Hsu [7] described a prey–predator model with two competing prey species. He analysed the model taking into account the logistic growth rate of both the prey and the interaction of the predator with both the prey as Holling type-I functional response. In this model the predator is fully dependent on both the prey. Cheng et al. [4] and Hsu [21] discussed a two dimensional general model with predator–prey interaction. They have taken some assumptions on specific growth rate of the prey in absence of predator and in predator functional response. They have also discussed that, there are other type of growth rate for the prey (except logistic growth rate) and functional response (except Holling type-I) which can satisfy the assumptions. But they have not considered the competition among predator. Again Xiao-Xiao and Hai-bin [22] showed that intra-specific interference coefficient is an important factor in prey–predator system.

Keeping the above in view, in this paper we have developed a general mathematical model consisting of two prey and one predator population. The growth rates of the two prey species and the functional response of the predators on the prey species are taken to be general non-linear functions. We assume that there is interspecific interaction between both the prey populations. We also assume that there is intraspecific interaction among the predator populations. Our main purpose is to discuss the effect of predation on two competing prey species in which the predator species are also influenced by the damage effect from its own species. By constructing a suitable Lyapunov's function, the local and global behaviour of the uniform steady state are investigated. Again in the numerical simulation part, we consider the interaction between prey–predator as the Holling type-II functional response. But in this function response, predator does not interfere with one another activity i.e. they are not competing with each other. But in experiment, observation is that feeding rate of predator decreases due to mutual interference of predator. So, in this case intra-specific interference coefficient is also taken into account in the modelling. Our model is more general than Hsu [7] and to the best of knowledge of authors, such general model does not appear in the literature.

2. Mathematical model

Consider an ecosystem where we wish to model the interaction of a predator with two competing prey species. Using Gauss model, the dynamics of the system can be governed by the following differential equations:

$$\frac{dN_1}{dt} = N_1 g_1(N_1, K_1) - b_{13} x p_1(N_1) - \alpha_{12} N_1 N_2, \tag{2.1}$$

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