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# Wave propagation in nonlocal microstretch solid

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ABSTRACT

This work investigates the possibility of plane waves propagating through an isotropic nonlocal microstretch solid of infinite extent. Five basic waves consisting of three longitudinal waves and two transverse waves may travel with distinct speeds. All these waves are frequency dependent and hence, dispersive in nature. The nonlocal parameter is present in the analytical expressions of phase speeds of all the existing waves. A comparison is also made to study the variation of phase speeds against nonlocal parameter. It is found that dispersion curves possess five branches: (a) a longitudinal acoustic branch, (b) a transverse acoustic branch, and (c) three optic branches. The reflection phenomenon of plane longitudinal wave incident at a stress free boundary surface of a nonlocal microstretch elastic half-space is studied and the formulae for various reflection coefficients are obtained. The variation of these reflection coefficients with angle of incidence has also been depicted graphically for a specific model.



## 1. Introduction

Theory of nonlocal elasticity has been developed by many researchers, e.g., Krumhansl [1] formulate constitutive relations using lattice theory, Kröner [2] formulated a continuum theory for elastic materials with long range cohesive forces, Eringen and Edelen [3], Edelen and Law [4], and Edelen et al. [5] developed the nonlocal elasticity theories characterized by the presence of nonlocality residuals of fields (like body force, mass, entropy, internal energy, etc.) and determined these residuals, along with the constitutive laws, by means of suitable thermodynamic restrictions. The concept of nonlocality has been extended to several other fields by Eringen [6–10], McCay and Narsimhan [11], Narsimhan and McCay [12]. The development of the nonlocal aspect of continuum mechanics has been nicely presented by Polizzotto [13]. Eringen [14] has introduced the theory of nonlocal polar elastic continua. He pointed out that in nonlocal theory of elasticity, the stress tensor at any reference point $\mathbf{x}$ within a continuous body depends not only on the strain at that point $\mathbf{x}$ but also significantly influenced by the strains at all other points $\mathbf{x}'$ of the continuous body. Thus, the nonlocal stress forces act as a remote action forces. These types of forces are frequently encountered in atomic theory of lattice dynamics. Within the context of nonlocal theory of elasticity, the length scale associated with nanostructures such as atomic distance between individual atoms can be represented by introducing small scale parameter in the constitutive equations. Such a nonlocal continuum mechanics is well established and has been applied to the problems of wave propagation. The application of nonlocal continuum mechanics for modeling and analysis of nanostructures has been made by several researchers, e.g., Narendar and Gopalakrishnan [15,16], Narendar et al. [17], Malagu et al. [18] etc.

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Keeping in view the importance of remote action forces, we have derived the constitutive relations for nonlocal microstretch elastic solid with the help of energy density function. Let us consider a nonlocal microstretch body occupying a volume $V$ bounded by a closed surface $S$. Since the force stress tensor, couple stress tensor and microstretch vector depend on the strains at all points of the body, therefore, these quantities are expressed in the form of integral over the entire volume of the body. The constitutive relations and the governing equations of small motion for linear isotropic nonlocal microstretch elastic solid are derived and presented in compact form. The field equations and constitutive relations are then used to investigate the propagation of plane waves in an infinite isotropic, nonlocal microstretch elastic solid. It is seen that there may exist five plane waves in this solid, comprising of three longitudinal and two transverse waves. Of the three longitudinal waves, one is an independent wave and other two are the two sets of coupled waves. Each set of these two coupled waves consists of a longitudinal displacement wave and a longitudinal microstretch wave. The two transverse waves are also coupled waves and each set of these two coupled waves consists of a transverse displacement wave and a transverse microrotational wave perpendicular to it. The phase speeds of all these waves are found to depend on nonlocal parameter as well as on the frequency parameter. Thus, the phase speeds of all the existing waves are influenced by nonlocality parameter and also dispersive in nature. Dispersion curves are found to possess five branches, namely, longitudinal acoustic branch, transverse acoustic branch and three optic branches. We have also obtained the formulae for amplitude ratios corresponding to various reflected waves when a set of coupled longitudinal waves is made to strike at the free boundary surface of a nonlocal microstretch half-space. The independent longitudinal wave and the two sets of coupled transverse waves existing in nonlocal microstretch elastic solid are the same as have been already encountered in nonlocal micropolar solid by Khurana and Tomar [19].

## 2. Constitutive relations and equations

Within the context of linear theory, the free energy density function $F$ is expressed as a quadratic symmetric function of the independent variables $\mathbf{x}$ and $\mathbf{x}'$. Following Eringen [20], the free energy density $F$ can be expressed as a symmetric polynomial in terms of basic variables $Y = \{T, \epsilon_{kl}, \gamma_{lk}, \gamma_k, \psi\}$ at $\mathbf{x}$ and $Y' = \{T', \epsilon'_{kl}, \gamma'_{lk}, \gamma'_k, \psi'\}$ at $\mathbf{x}'$ as follows:

$$2F = 2F_0 - \frac{2\rho_0}{T_0}CTT' - C_1(T'\psi + T\psi') - D_k(T'\gamma_k + T\gamma'_k) - A_{kl}(T'\epsilon_{kl} + T\epsilon'_{kl}) - B_{kl}(T'\gamma_{kl} + T\gamma'_{kl}) + U,$$

where $T$ is temperature, $\epsilon_{kl}$ is the relative distortion tensor, $\gamma_{kl}$ is the curvature tensor, $\gamma_k$ is the microstretch gradient and $\psi$ is scalar microstretch. A prime on the symbols denotes their dependence on $\mathbf{x}'$. The quantities $\epsilon_{kl}$, $\gamma_{kl}$ and $\gamma_k$ are given by:

$$\epsilon_{kl} = u_{l,k} - \epsilon_{klm}\phi_m, \quad \gamma_{kl} = \phi_{k,l}, \quad \gamma_k = \psi_{,k}, \tag{1}$$

and $C$, $C_1$, $D_k$, $A_{kl}$, $B_{kl}$ are the material moduli. The quantities $F_0$, $\rho_0$ and $T_0$ are constants and may be defined as energy density in the natural state of the body, rest density and ambient temperature, respectively. The symbol $U$ denotes the strain energy density function, given by:

$$\begin{aligned}2U = {}& \frac{\rho_0}{T_0}CTT' + C^S\psi\psi' + C^S_k(\psi'\gamma_k + \psi\gamma'_k) + A^S_{kl}(\psi'\epsilon_{kl} + \psi\epsilon'_{kl})\\ &+ B^S_{kl}(\psi'\gamma_{kl} + \psi\gamma'_{kl}) + C^S_{kl}\gamma'_k\gamma_l + A^S_{klm}(\gamma'_k\epsilon_{lm} + \gamma_k\epsilon'_{lm}) + B^S_{klm}(\gamma'_k\gamma_{lm} + \gamma_k\gamma'_{lm})\\ &+ A_{klmn}\epsilon'_{kl}\epsilon_{mn} + B_{klmn}\gamma'_{kl}\gamma_{mn} + C_{klmn}(\epsilon'_{kl}\gamma_{mn} + \epsilon_{kl}\gamma'_{mn}),\end{aligned}$$

where $C^S, C^S_k, A^S_{kl}, B^S_{kl}, C^S_{kl}, A^S_{klm}, B^S_{klm}, A_{klmn}, B_{klmn}$ and $C_{klmn}$ are the material moduli.

Note that the expression of free energy function $F(Y, Y')$ has been obtained by expanding it in the neighborhood of a natural state into a Taylor series and omitting the terms having powers higher than two.

All the material moduli are functions of $\mathbf{x}, \mathbf{x}'$ and follow the symmetric regulations, e.g.,

$$\begin{aligned}&C(\mathbf{x},\mathbf{x}') = C(\mathbf{x}',\mathbf{x}), \quad A_{kl}(\mathbf{x},\mathbf{x}') = A_{kl}(\mathbf{x}',\mathbf{x}) = A_{lk}(\mathbf{x},\mathbf{x}'),\\ &A^S_{klm}(\mathbf{x},\mathbf{x}') = A^S_{klm}(\mathbf{x}',\mathbf{x}) = A^S_{lkm}(\mathbf{x},\mathbf{x}'), \quad A_{klmn}(\mathbf{x},\mathbf{x}') = A_{klmn}(\mathbf{x}',\mathbf{x}) = A_{mnkl}(\mathbf{x},\mathbf{x}'), \text{ etc.}\end{aligned}$$

Following Eringen [20], the constitutive relations are obtained from the relation:

$$J = \int_V \left[\frac{\partial F}{\partial Y} + \left(\frac{\partial F}{\partial Y'}\right)^S\right] dV',$$

where the set $J = \{-\rho_0\eta, t_{kl}, m_{kl}, m_k, s - t\}$ is an ordered set with set $Y$ and $\eta$ is the entropy density. Thus, the force stress tensor ($t_{kl}$), couple stress tensor ($m_{kl}$), microstretch vector ($m_k$) and the quantities $\eta$ and $s - t$ are given by:

$$\begin{aligned}\rho_0\eta = {}& \int_V \Big[\frac{\rho_0}{T_0}C(\mathbf{x},\mathbf{x}')T(\mathbf{x}') + C_1(\mathbf{x},\mathbf{x}')\psi(\mathbf{x}') + D_k(\mathbf{x},\mathbf{x}')\psi_{,k}(\mathbf{x}')\\ &+ A_{kl}(\mathbf{x},\mathbf{x}')\epsilon_{kl}(\mathbf{x}') + B_{kl}(\mathbf{x},\mathbf{x}')\gamma_{kl}(\mathbf{x}')\Big] dV(\mathbf{x}'),\end{aligned}$$

$$t_{kl} = \int_V [-A_{kl}(\mathbf{x},\mathbf{x}')T(\mathbf{x}') + A^S_{kl}(\mathbf{x},\mathbf{x}')\psi(\mathbf{x}') + A^S_{mkl}(\mathbf{x},\mathbf{x}')\psi_{,m}(\mathbf{x}')$$