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Formulation of three-dimensional equations of motion for train–slab track–bridge interaction system and its application to random vibration analysis

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ABSTRACT

This study presents the formulation of three-dimensional equations of motion for a train–slab track–bridge interaction system and its application to random vibration analysis using the finite element and pseudo-excitation methods. In this study, a train, slab track, and bridge are regarded as an integrated system, each vehicle is modeled as a four-wheelset mass-spring-damper system with a two-layer suspension system at 23 degrees of freedom, and the rail, slab, girder, and pier are modeled as elastic Bernoulli–Euler beams connected with each other by discrete or continuous spring and damper elements. Three-dimensional equations of motion for the entire system are derived using the energy principle. Dynamic contact forces between moving vehicles and rails are considered as internal forces, and thus, the excitation vectors of load between a wheel and rail, induced by a vehicle's weight and random track irregularities, are easily formulated using the pseudo-excitation method. These equations can be solved by a step-by-step integration method to simultaneously obtain the random dynamic responses of the system. The three-dimensional random vibration characteristics of the system are investigated using an example of a nine-span simply supported beam bridge on which a train consisting of 8 cars travels.

1. Introduction

The dynamic responses of railway track/bridge structures to moving loads exerted by trains have been widely investigated by researchers. However, studies concentrating on train–track/bridge interaction were restricted to two-dimensional aspects of the train–track/bridge system and aimed at analyzing vertical vibration within the system [1–10]. Only a relatively small amount of research has focused on the three-dimensional aspects of the train–track/bridge system [11–15], and such studies have either neglected to include the direct involvement of the track system with a moving train or only the conventional ballasted track has been considered. Since the track system is a flexible medium that vibrates with the train and bridge, it can have a serious effect on the extent of interaction between the two subsystems, particularly in modern high-speed railways [16] in which slab ballastless tracks (or slab tracks) are widely adopted [17,18]. To date, there is little literature

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available that discusses the three-dimensional dynamic responses of the train–slab track–bridge interaction system under track irregularities.

It is well known that [16] track irregularities have a random nature and are one of the most important factors that can amplify the vibration responses of the train–track/bridge interaction system. In most pioneering research, track irregularities have usually only been treated as time-history samples for computing dynamic responses. However, because of the randomness of track irregularities, each result in this respect should be regarded as a unique case that can have a sequence of possible outcomes. With an increase in the speed of trains, random vibrations of the train–track/bridge interaction system are receiving increased attention, and these are predominantly analyzed using the Monte Carlo method (MCM) [19,20], where the statistical characteristics of responses are computed according to response samples from numerous time-history samples of track irregularities. For example, Xia and Zhang [20] computed the maximum responses of a one-span simply supported beam bridge traversed by a train traveling at a constant of speed 200 km/h; the study contained 20 samples of random track irregularities. Results showed that the coefficient of variations relating to the vertical acceleration of the bridge at the midpoint and that of car body acceleration approached 13.09% and 21.25%, respectively. A sufficient number of samples are required to ensure the reliability of simulations; however, the use of a high number of samples involves an unacceptable amount of computer time. Therefore, algorithms with greater efficiencies and accuracies such as the pseudo-excitation method (PEM) are required in the analysis of random vibrations in the train–slab track–bridge interaction system [21–25].

It is also necessary to consider the slab track system, which usually consists of a precast concrete slab (slab), cement asphalt mortar (CAM), and a cut in situ concrete base (base). It is quite different from the conventional ballasted track system that is usually composed of precast concrete (or wood) sleepers and a ballast layer, and thus, the interaction characteristics of the train–slab track–bridge system significantly vary from those of the train–ballasted track–bridge system.

In this study, there are two main aims. The first is to derive the three-dimensional equations of motion for the interaction system consisting of a moving train, slab track, and bridge; the second is to investigate the random vibration characteristics of the train–slab track–bridge interaction system. Compared with previously presented theories [7,25,26], enhancements introduced in this study for the three-dimensional train–slab track–bridge interaction model (not for the vertical train–slab track–bridge interaction model, vertical train–ballasted track–bridge interaction model, three-dimensional train–ballasted track–bridge interaction model, or three-dimensional train–bridge interaction model) deliver the possibility of considering random excitations between a wheel and rail (not between a wheel and girder) and thus permit a more realistic analysis. The following are presented in this paper. First, assumptions made for modeling the three-dimensional train–slab track–bridge interaction system are summarized. Second, the equations of motion for the major components of the system, i.e., the vehicle, rail, slab, girder, and pier, are derived in detail using the energy principle [26]. Moreover, using the information presented, the equations of motion for the entire train–slab track–bridge interaction system are then assembled. Third, by considering time lags between wheels, the effects of track irregularities are regarded as a series of uniformly modulated, multi-point, different-phase random excitations, and excitations between a wheel and rail caused by the random track irregularities are then transformed into a series of deterministic pseudo-harmonic excitation vectors using PEM [25], thereby enabling the random vibration responses of the train, slab track, and bridge to be obtained using a step-by-step integration method. Finally, taking a nine-span, simply supported beam bridge traversed by a train consisting of 8 cars as an example, the reliability and efficiency of PEM for calculating random vibration responses are verified through comparison with MCM, and the random vibration characteristics of the train–slab track–bridge interaction system are analyzed based on solutions obtained by PEM.

2. Models of train, slab track, and bridge

2.1. Model of train

Fig. 1 shows a train consisting of a series of four-wheelset vehicles (rear and front cars numbered 1 and 2, respectively, with N_v trailer cars numbered 1, 2, … , N_v from left to right) moving at a constant speed, v. The railway bridge and two approach subgrades are modeled using a slab track structure resting on a series of simply supported beams.

Each trailer car of the train is modeled as having a mass-spring-damper system consisting of one car body, two bogies, four wheelsets, and a two-stage suspension system. As shown in Fig. 1, the car body rests on the front and rear bogies, each of which is then supported by two wheelsets. The car body is modeled as a rigid body with mass, m_c, and three moments of inertia, I_{cx}, I_{cy}, and I_{cz}. Similarly, each bogie is considered as a rigid body with mass, m_t, and three moments of inertia, I_{tx}, I_{ty}, and I_{tz}. In addition, each wheelset is considered as a rigid body with mass, m_w, and two moments of inertia, I_{wx} and I_{wz}. The secondary suspension between the car body and each bogie is characterized using a three-dimensional system of springs with stiffnesses k_{sx}, k_{sy}, and k_{sz}, and dampers with damping coefficients c_{sx}, c_{sy}, and c_{sz}. Likewise, the springs and shock absorbers in the primary suspension for each wheelset are characterized as k_{px}, k_{py}, and k_{pz}, and c_{px}, c_{py}, and c_{pz}, respectively. By neglecting longitudinal displacements, the motions of the jth trailer car body, with respect to its center of gravity, can be described by y_{cj}, z_{cj}, θ_{cj}, φ_{cj}, and ψ_{cj}. Similarly, the motions of both the rear and front bogies of the jth trailer car can be described, respectively, by y_{t1j}, z_{t1j}, θ_{t1j}, φ_{t1j}, and ψ_{t1j} and y_{t2j}, z_{t2j}, θ_{t2j}, φ_{t2j}, and ψ_{t2j}. Furthermore, the motions from left to right of the hth ($h = 1$–4) wheelset of the jth trailer car can be described by y_{whj}, z_{whj}, θ_{whj}, and ψ_{whj}, respectively. Therefore, the total number of degrees of freedom (DOFs) for each trailer car is 31. However, in this paper it