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Non-local behavior of two collinear mixed-mode limited-permeable cracks in a functionally graded piezoelectric medium

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ABSTRACT

In this paper, the problem of two collinear mixed-mode limited-permeable cracks embedded in an infinite medium made of a functionally graded piezoelectric material (FGPM) with crack surfaces subjected to electro-mechanical loadings is investigated. Eringen's non-local theory of elasticity is adopted to formulate the governing electro-elastic equations. The properties of the piezoelectric material are assumed to vary exponentially along a perpendicular plane to the crack. Employing the Fourier transform, the mixed-boundary value problem is converted into three integral equations for each crack, with the unknown variables being the jumps of mechanical displacements and electric potentials across the crack surfaces. To solve the integral equations, the unknowns are directly expanded as a series of Jacobi polynomials, and the resulting equations solved using the Schmidt method. In contrast to the classical solutions based on the local theory, it is found that no mechanical stress and electric displacement singularities are present at the crack tips when nonlocal theory is employed to investigate the problem. A direct benefit is the ability to use the calculated maximum stress as a fracture criterion. The primary objective of this study is to investigate the effects of the interaction of two cracks, material gradient parameter describing functionally graded piezoelectric materials and lattice parameter on the mechanical stress and electric displacement field near crack tips.

1. Introduction

Discovered as early as 1880 by Pierre and Jacques Curie, piezoelectricity can be defined as the linear electromechanical interaction between the mechanical and electrical states of crystals devoid of a center of symmetry [1]. Piezoelectric materials exhibit the piezoelectric effect, that is, electric polarization is induced in the material on application of mechanical loads and vice-versa [2]. Indeed, it is this fundamental electro-mechanical coupling phenomenon that endows piezoelectric materials with several desirable properties that can be exploited in the design of a wide range of engineering applications like transducers, sensors and actuators [3]. However, a major shortcoming of piezoelectric ceramics is that they are extremely

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brittle with a propensity to develop cracks due to stress concentrations induced as a consequence of both mechanical and electrical loadings [4].

In recent years, applications that require piezoelectric ceramics are geared towards larger components and severe loading conditions. Therefore reliability and durability are of paramount importance. For instance, piezoelectric biomorphs which are an important class of piezoelectric devices show tendency for stress concentrations and internal debonding when subject to repeated cycles of electro-mechanical loadings. This is because the biomorphs are generally made of two thin piezoelectric elements bonded by using adhesives such as epoxy resins. In the presence of repeated variations of temperature and loadings, the bonding agents can develop cracks reducing the reliability of these devices [5]. A class of materials which could potentially alleviate the problem of internal debonding and stress concentration in conventional piezoelectric materials are functionally graded materials (FGMs). Over the past two decades FGMs have already shown great promise to be used as an alternative to conventional homogenous coatings [6]. These materials are generally composed of at least two-phase inhomogeneous particulate composites synthesized in such a manner that the volume fractions of the constituents vary continuously along any desired spatial direction, resulting in materials having smooth variation of mechanical properties. Therefore, any material property discontinuities that exist maybe suitably eliminated when FGMs are used as coating material, for example, ceramic coating on metal surfaces, leading to a significant reduction in residual and thermal stress concentration and increased resilience to fracture [6]. Therefore, the concept of FGMs can be extended to piezoelectric materials to synthesize functionally graded piezoelectric materials (FGPMs) that exhibit improved reliability. The FGM based piezoelectric devices may be completely made of FGPM alone or the FGPM layer could be used instead of the bonding agent eliminating any internal seams or boundaries, providing for a smooth variation in material properties [5]. For instance, in a study by Wu et al. [7], it was shown that by introducing a controlled functional gradient of piezoelectric activity in a piezoelectric ceramic such as lead zirconate-titanate (PZT), stress peaks can be avoided in the PZT bars. For reliable service lifetime prediction of FGPM based devices, it is imperative to investigate and understand the fracture behavior of FGPMs and their effects on the electro-mechanical response.

Most of the previous investigations into the fracture of FGPMs adopt classical continuum mechanics based on the local theory. Another point easily noticed is that these studies are almost exclusively devoted to the analysis of anti-plane mode III crack problems. We summarize a few relevant studies in the following paragraph.

Li and Weng [5] solved the anti-plane crack problem in a transversely isotropic infinitely long functionally graded piezoelectric strip. Employing Yoffe's model, the moving mode III permeable crack problem in FGPMs was studied by Jin and Zhong [8]. Ueda [9] studied the problem of a crack embedded in a functionally graded piezoelectric strip bonded to two elastic surface layers, which are subjected to anti-plane mechanical loading and in-plane electric loading. Wang [10] examined the mode III crack problem in FGPMs for cases of a single crack and a series of collinear cracks embedded in the medium. Ma et al. [11] investigated the electro-elastic behavior of a Griffith crack in an infinitely long functionally graded piezoelectric strip under anti-plane mechanical loading and in-plane electrical loading. Considering both electrically impermeable and permeable crack surfaces Chue and Ou [12] studied the problem of a mode III crack oriented perpendicular to the interface formed by bonding two functionally graded piezoelectric half-planes. As an extension of this work, Ou and Chue [13] later analyzed the same mode III crack problem with two cracks embedded in each of the two bonded functionally graded piezoelectric half-planes. Further, the same group of Ou and Chue [14] examined the mode III eccentric crack located in a single functionally graded piezoelectric strip considering both permeable and impermeable crack surfaces. Hsu and Chue [15] solved the mode III fracture problem of an arbitrarily oriented crack in functionally graded piezoelectric strip bonded to a homogenous piezoelectric half plane. Chen and Chue [16] investigated the mode III fracture problem of a cracked functionally graded piezoelectric surface layer bonded to a cracked functionally graded piezoelectric substrate.

A survey of the literatures shows that as opposed to mode III crack problems in FGPMs, studies concerning mode I, mode II and mixed mode crack problems are far and few between. Wang and Noda [17] studied the thermopiezoelectric fracture problem of a functionally graded piezoelectric layer bonded to a metallic substrate under in-plane loadings. Chen et al. [18] obtained the Mode-I crack transient response of an infinite functionally graded piezoelectric medium subject to in-plane mechanical and electric impact. Ueda [19] analyzed the mixed mode dynamic fracture problem for functionally graded piezoelectric strip containing a parallel crack under in-plane mechanical and electric impact loadings. Recently, Zhou and Chen [20] examined the interaction of two parallel mode I limited-permeable cracks in FGPMs.

Common to all aforementioned studies is the use of classical continuum mechanics techniques based on the local assumption to investigate crack problems in FGPMs. According to local elasticity theory, the state of stress at a specific point in the material depends only on the state of strain at the same point. Contrary to physical reasoning, the application of local elasticity theory invariably leads to stress singularities at the crack tips. A major issue here is that stress at the crack tips is indeterminate and thus there a fracture criterion based on maximum stress is not easy to establish.

Different from classical local elasticity theory is the nonlocal elasticity theory which attempts to develop the constitutive relationships without foregoing the microstructure of the material. Nonlocal continuum mechanics initiated by Eringen [21], Eringen and Adelen [22] is based on the nonlocal elasticity model, where the state of stress at a given point is a function of the strain states at all points in the material. The nonlocal theory was employed to great success by Eringen [23,24], and Eringen et al. [25] to investigate the stress near the tip of a sharp line crack in an isotropic elastic plate. Unlike classical (or local) elasticity theory, it was shown that the stress field calculated using nonlocal theory does not contain any singularities