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Multi-field coupling static bending of a finite length inhomogeneous double-layered structure with inner hollow cylinder and outer shell

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ABSTRACT

This study deals with the static bending of a finite length double-layered structure under radially symmetric mechanic and thermal loadings. The double-layered hollow structure is constructed of inner thermal barrier cylinder and outer FGPM(functionally graded piezoelectric material)cylindrical shell. The inner hollow cylinder is subjected to symmetric mechanic and thermal loadings, the outer shell is under an extra electric field. The material parameters of the double-layered structure are assumed to be inhomogeneous, or, rather, follows a power-law distribution. The inner thermal barrier cylinder is solved by the finite difference method, and the outer cylindrical shell is solved by using the analytical method. In the examples, numerical results separately show the effect of material properties, geometric shapes, mechanical and temperature loads on the double-layered structure. The distributions of material properties and the thickness ratio have a great influence on displacement and stress of the double-layered structure. Results in this study may be useful for the optimal design of FGPM cylindrical shell with thermal barrier layer.

1. Introduction

Thermal barrier material is a kind of material or composite material which has a good performance of adiabaticity and can shield effects of heat flow. The failure of thermal barrier coatings under the environment of high temperature has been widely concerned. Evans et al. [1] investigated the failure mechanisms of thermal barrier coatings with diffusion aluminide bond coatings, they [2] also proposed a protocol for validating models of the cyclic undulation of thermally grown oxides. The design, deposition, and internal stresses of a thick multilayer thermal barrier coating had been studied by Samadi [3]. Azadi [4] presented a new thermo-mechanical fatigue lifetime prediction model for the A356.0 aluminum alloy with thermal barrier coatings. Stress analysis of thermal barrier coating system subjected to out-of-phase thermo-mechanical loadings considering roughness and porosity effect were presented by Rezvani Rad et al. [5,6], and they studied the effects of preheating temperature and cooling rate on two-step residual stress in thermal barrier coatings considering real roughness

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and porosity effect. Moridi et al. [7] also presented investigated a thermo-mechanical stress analysis of thermal barrier coating system considering thickness and roughness effects.

In practical applications, thermal barrier material can be found in pipeline structures to constitute a double-layered cylinder. Hollow cylinders are a kind of typical structures in engineering applications, such as piping system and pressure vessels. Piping systems, as a convenient, reliable and economical means of transportation, are often used to transport petroleum, natural gas, coal liquid, etc. To find a kind of structures which are more adaptable to special working environment, finite length cylinder has got the extensive research. Shao [8] proposed the analytical solutions about stresses in functionally graded circular hollow cylinder with finite length. By using a multi-layered approach based on the theory of laminated composites, he [9] also researched the mechanical and thermal stresses of a functionally graded circular hollow cylinder with finite length. Jabbari et al. [10] put forward the solution of axisymmetric mechanical and thermal stresses in thick short length FGM(functionally graded material) cylinders by means of the generalized Bessel function and Fourier series. Asgari [11–13] did a lot of work about two-dimensional functionally graded thick hollow cylinder with finite length, these articles are about dynamic analysis, transient heat conduction and transient thermal stresses, respectively.

Double-layered and multi-layered hollow cylinders are also extensively researched in recent years. Wang and Gong [14] obtained the elastic-dynamic solution for multilayered cylinders subjected to axisymmetric dynamic loads by means of the finite Hankel transform and Laplace transform. By using the expansion of transient wave functions in a series of eigenfunctions, Yin and Yue [15] studied the transient plane-strain response of multilayered cylinders due to axisymmetric impulse. Thermal buckling for local delamination near the surface of laminated cylindrical shells and delaminated growth were presented by Wang and Dai [16], they [17] also studied magneto thermodynamics stress and perturbation of magnetic field vector in a hollow cylinder. The principle of superposition was successfully applied by Wang et al. [18] to analyze the axisymmetric plane strain dynamic problems for multilayered orthotropic piezoelectric infinite hollow cylinder, and then Wang et al. [19] studied the transient responses in a two-layered piezoelectric composite hollow cylinder in the state of axisymmetric plane strain. Yoshinobu [20] investigated transient piezo thermoelasticity of a two-layered composite hollow cylinder constructed of isotropic elastic and piezoelectric layers due to asymmetrical heating, the transient two-dimensional temperature is analyzed by the method of Laplace transformation and the exact solutions for piezoelectric hollow cylinder and isotropic hollow cylinder had been obtained. Fesharaki et al. [21] examined the general theoretical analysis for a hollow cylinder made of functionally graded piezoelectric material subjected to two-dimensional electromechanical load with the separation of variables method and complex Fourier series. Application of a hybrid meshless technique for natural frequencies analysis in functionally graded thick hollow cylinder subjected to suddenly thermal loading was presented by Hosseini [22]. Dai et al. [23,24] studied the dynamic thermoelastic behavior of a long functionally graded hollow cylinder and a double-layered hollow cylinder with an FGM layer by the finite difference method (FDM), Newmark method and iterative method. By the same method, Dai et al. [25] and Xie [26] studied the thermo viscoelastic dynamic behavior of a double-layered hollow cylinder under thermal shocking and thermoelastic dynamic behaviors of a FGM hollow cylinder under non-axisymmetric thermo-mechanical loads respectively. Based on the infinitesimal theory of electro magneto thermoelasticity, Dai et al. [27] put forward an analytical solution for electro magneto thermoelastic behaviors of a functionally graded piezoelectric hollow cylinder.

Moreover, the cylindrical shell structure is also widely used in piping systems. Many studies focused on buckling behaviors of the FGM cylindrical shell. Linear thermal buckling behaviors of the FGM cylindrical shell were investigated in literatures (Shahsiah and Eslami [28]; Wu et al. [29]; Kadoli and Ganesan [30]), in which temperature fields were assumed to be uniform or gradient through the shell thickness. Sheng and Wang [31] present the report of an investigation into thermoelastic vibration and buckling characteristics of the functionally graded piezoelectric cylindrical shell. Zhang and Li [32] carried out buckling and vibration analysis of functionally graded magneto-electro-thermo-elastic circular cylindrical shell. Shen [33] presented thermal post-buckling analysis for an FGM cylindrical shell under uniform temperature field and heat conduction shell theory based on higher order shear deformation shell theory. Moreover, Zhao et al. [34] and Zhao and Liew [35,36] used the element-free kp-Ritz method for analysis of the thermal and mechanical buckling of functionally graded cylindrical shell structures. Dai et al. [37–40] also did a lot of work about the buckling and post-buckling analyses of an FGM cylindrical shell.

However, to our knowledge, researches on the double-layered structure mentioned in this paper are not comprehensive. Therefore, in this paper, a finite length double-layered structure under radially symmetric mechanic and thermal loadings is investigated. The inner thermal barrier cylinder is solved by the FDM, and for outer FGPM(functionally graded piezoelectric material) cylindrical shell, an analytical-solution is achieved. Influence of the material properties, geometric shapes and external load are considered in numerical examples.

2. Material properties and structural model

Consider a double-layered hollow structure with finite length L. The outer layer is an FGPM cylindrical shell and the inner layer is a thermal barrier cylinder made of inhomogeneous material. The inner layer is subjected to an axisymmetrical, uniformly distributed dynamic load q and temperature distribution $T(r)$ along the radial thickness, while the outer layer is under an electric field excitation. The thickness, inner and outer radii of the hollow cylinder are respectively denoted by t, a and b, and a cylindrical coordinates (r, θ, x) is established as shown in Fig. 1. For cylindrical shell, the radii of middle surface and the thickness are respectively denoted by $R(R = b + h)$ and h. Orthogonal curvilinear coordinate system (x, y, z)