Applied Mathematical Modelling 40 (2016) 750–762

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

Numerical analysis of the interactions between laser and soft tissues using generalized dual-phase lag equation

M. Jasiński*, E. Majchrzak, L. Turchan

Institute of Computational Mechanics and Engineering, Silesian University of Technology, Konarskiego 18A, 44-100 Gliwice, Poland

ARTICLE INFO

Article history:
Received 24 April 2015
Revised 8 October 2015
Accepted 15 October 2015
Available online 2 November 2015

Keywords:
Bioheat transfer
Generalized dual-phase lag equation
Arrhenius integral
Finite difference method

ABSTRACT

In the paper a numerical analysis of thermal processes proceeding in a soft tissue subjected to a laser irradiation is presented. The transient bioheat transfer in the 3D domain considered is described by the generalized dual-phase lag equation. The internal heat source resulting from the laser irradiation based on the solution of the diffusion equation is taken into account. The process of the thermal destruction of the tissue is also analyzed. At the stage of numerical realization the explicit scheme of finite difference method is used. In the final part of the paper the results of computations for different variants of tissue porosity are shown.

1. Introduction

Modeling of physical processes proceeding in biological tissues subjected to a laser beam can be divided into three steps, namely the modeling of laser energy deposition, the temperature distribution and the damage in laser irradiated tissue [1].

To describe the light propagation in biological tissues the different mathematical models can be used [2–7]. The most fundamental approach, which is difficult from the mathematical point of view, is the so-called analytical theory which starts from the Maxwell equations. On the other hand, the transport theory concerns directly the transport of light through absorbing and scattering media and in this case the radiative transport equation should be taken into account, e.g. [8–12]. To solve this equation several modifications of the discrete ordinates method (DOM) and statistical Monte Carlo (MC) methods are widely used, e.g. [13,3,14,15]. In some cases it is possible to approximate the light transport using the diffusion equation [16–20]. As the scattering generally dominates over the absorption in soft tissues for wavelengths between 650 and 1300 nm, so in this paper the diffusion approximation is applied.

The Pennes equation [21] is one of the earliest bioheat transfer equations which describes the temperature distribution in the living tissues and for the constant thermophysical parameters it has a form:

$$\rho c \frac{\partial T}{\partial t} = \lambda \nabla^2 T + Q, \tag{1}$$

where c [J/(kg K)] is the specific heat of tissue, ρ [kg/m^3] is the density, λ [W/(m K)] is the thermal conductivity, $T = T(\mathbf{x}, t)$ is the tissue temperature ($\mathbf{x}$ denotes spatial coordinates, t is the time).

The capacity of internal heat sources $Q = Q(\mathbf{x}, t)$ [W/m^3] can be written in the following form:

$$Q = wc_b(T_b - T) + Q_m + Q_{ex}, \tag{2}$$

* Corresponding author. Tel.: +48 32 237 12 04; fax: +48 32 237 12 82.
E-mail address: marek.jasinski@polsl.pl (M. Jasiński).

http://dx.doi.org/10.1016/j.apm.2015.10.025
S0307-904X(15)00683-6/

where w [kg/(m^3 s)] is the blood perfusion rate, c_b is the specific heat of blood, T_b is the arterial blood temperature, Q_m is the metabolic heat source and $Q_{ex} = Q_{ex}(\mathbf{x}, t)$ is the source function associated with the external heating of tissue. The Pennes bioheat equation is based on the classical Fourier's law of heat conduction which assumes that the thermal disturbance propagates with an infinite speed. So far, it is the most frequently used model to determine the temperature distribution in biological tissues, e.g. [18,22–29].

Heterogeneous structure of biological tissues is taken into account in the other models of bioheat transfer, for example in the hyperbolic thermal wave model (Cattaneo–Vernotte equation) or the dual-phase lag model (DPLM). In the Cattaneo–Vernotte equation [30,31]:

$$c\rho\left(\frac{\partial T}{\partial t} + \tau_q \frac{\partial^2 T}{\partial t^2}\right) = \lambda \nabla^2 T + Q + \tau_q \frac{\partial Q}{\partial t}, \tag{3}$$

the relaxation time τ_q [s] is the delay in establishing the heat flux vector, in others words, the temperature gradient always precedes the heat flux vector.

In the dual-phase lag equation [32]:

$$c\rho\left(\frac{\partial T}{\partial t} + \tau_q \frac{\partial^2 T}{\partial t^2}\right) = \lambda \nabla^2 T + \lambda \tau_T \frac{\partial}{\partial t}\left(\nabla^2 T\right) + \tau_q \frac{\partial Q}{\partial t}, \tag{4}$$

both the heat flux and temperature gradient are delayed. The delay of temperature gradient is called the thermalization time τ_T [s].

As can be seen, for $\tau_T = 0$ the DPLM is simplified to the Cattaneo–Vernotte equation and for $\tau_T = \tau_q = 0$ to the Pennes equation. In the discussed models the appropriate estimation of the delay times is the significant problem. Values of these parameters are usually determined experimentally [33–35] and they are ranged from 0.01 to 32 s.

In recent years, the thermal wave model and the dual-phase lag equation are often used to calculate the temperature distribution in biological tissues, e.g. [11,12,36–41].

The last step in the analysis of tissue heating process is to estimate the degree of its destruction. For this purpose, the so-called Arrhenius integral is most frequently applied, although other models are also used, for example the thermal dose parameter [42,43]. Arrhenius scheme assumes the exponential dependence between the temperature and the degree of tissue destruction, so the following integral is taken into account [26,5]:

$$\Theta(\mathbf{x}) = P \int_0^{t^F} \exp\left(-\frac{E}{R_g T(\mathbf{x}, t)}\right) \mathrm{d}t, \tag{5}$$

where P [1/s] is the pre-exponential factor, E [J/mol] is the activation energy, R_g [J/(mol K)] is the universal gas constant, $T(\mathbf{x}, t)$ [K] is the tissue temperature, t denotes time and $[0, t^F]$ is the time interval under consideration.

A value of damage integral $\Theta(\mathbf{x}) = 1$ corresponds to a 63% probability of cell death at a specific point $\mathbf{x}$, while $\Theta(\mathbf{x}) = 4.6$ corresponds to 99% probability of cell death at this point [44]. Besides the basic information on whether the tissue was destructed or not (in accordance with the necrosis criterion, this means $\Theta(\mathbf{x}) \geq 1$), one can deduce the time after which the tissue damage is to occur.

The tissue injury integral basically refers only to the irreversible tissue damage, however, there are models which allow to take into account the withdrawal of tissue injury in the case of temporary, small local increasing of temperature [27]. Through the approach presented in [27] it is possible to determine the time after which the thermal lesion is formed.

The purpose of this paper is to analyze the phenomena occurring in the laser-treated soft tissue, in which the heat conduction in the tissue is described by the generalized dual-phase lag model [45]. In this model, based on the theory of porous media the phase lag times are expressed in terms of the properties of blood and tissue, interphase convective heat transfer coefficient and the blood perfusion rate. Using this model, for different levels of tissue perfusion the degree of their destruction can be compared. Thus, the paper consists of the following parts. At first, the generalized dual-phase lag equation (GDPLE) is presented and the problem analyzed is formulated. Next, the way of GDPLE solution basing on the explicit scheme of the finite difference method is described and the examples of computations are shown. In the final part of the paper the conclusions are formulated.

2. Generalized dual-phase lag equation

Heterogeneous structure of biological tissue can be described in several ways. One of them is to divide the tissue into two regions: the vascular region (blood vessels) and the extravascular region (tissue) [46,47]. Then, the porosity ε is defined as the ratio of blood volume to the total volume and two equations describing the temperature distribution in the tissue T_t and blood vessels T_b are considered, namely

$$(1-\varepsilon)\rho_t c_t \frac{\partial T_t}{\partial t} = (1-\varepsilon)\lambda_t \nabla^2 T_t + \alpha A(T_b - T_t) + w c_b (T_b - T_t) + (1-\varepsilon) Q_{mt} + (1-\varepsilon) Q_{ex} \tag{6}$$

and

$$\varepsilon \rho_b c_b \left(\frac{\partial T_b}{\partial t} + \mathbf{u} \cdot \nabla T_b\right) = \varepsilon \lambda_b \nabla^2 T_b + \alpha A(T_t - T_b) + w c_b (T_t - T_b) + \varepsilon Q_{mb} + \varepsilon Q_{ex}, \tag{7}$$