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Planar locomotion of a vibration-driven system with two internal masses

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ABSTRACT

A vibration-driven system is modelled to achieve the expected planar locomotion in the present paper. The system is composed of a main rigid body and two internal and movable masses. Coulomb dry friction and nonholonomic constraint of the body are considered to model stick-slip effect and to ensure the motion without sideslip. It is seen that driving periodically the two masses in two orthogonal directions leads to the expected planar locomotion. Correspondingly, the translational and rotary velocities of the system are analytically obtained. As a result, the system can move not only along oblique line in any given slope but also along folding lines derived from the oblique lines in the different slopes.

1. Introduction

The present paper deals with the planar locomotion of a kind of vibration-driven system. It is well known that most locomotion systems are propelled by protruding components, like wheels and legs which severely reduce the adaptability of their application, especially in restricted environments. Vibration-driven systems, however, are a new kind of movable dynamical systems propelled by the periodic motion of internal masses or the periodic deformation of their bodies. Animals like earthworms and snakes are good representatives that employing the vibration-driven system to locomote in various media. Learning from the earthworms and the snakes to develop bio-inspired locomotion robots is currently an attractive research field [1,2]. Compared to the conventional legged and wheeled robots, due to the relatively simpler structure, stronger controllability and higher miniaturization potential, these earthworm-like or snake like robots are promisingly to be used in disaster rescue, pipeline inspection and minimally invasive surgery.

The vibration-driven locomotion systems have been widely studied in recent years from many aspects. Among them, Chernousko [3–7] took the first step in setting up the theoretical formula of the motion of a rigid box with movable internal mass. Anisotropic Coulomb dry friction or anisotropic viscous friction is assumed to act between the system and the rough environment. The anisotropism of the friction forces are a key factor of the system's directed locomotion. Two types of periodic motion are assigned to the internal mass, one is velocity-controlled motion, and the other is acceleration-controlled motion. With these two controlling modes, the vibration driven system is able to perform effective rectilinear motion. Optimization is carried out on both of these two internal motions with the same objective–maximizing the average velocity of the system as a whole. Experimental verification of the theoretical analysis is another important aspect [8,9]. Li and Furuta set up a pendulum-driven

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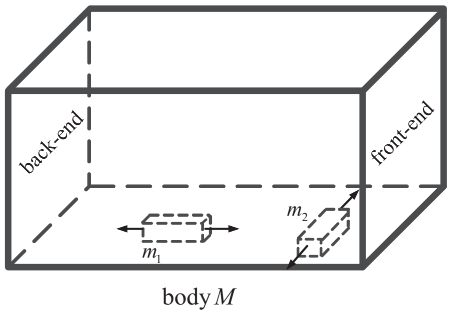

Fig. 1. 3-D scheme of the modelled system.

cart and fabricated out the Capsubot, which manifest the effectiveness of the vibration-driven system in terms of rectilinear motion.

Focusing on the approximate steady-state motion, stick-slip motion, and motion optimization, the state of the art of the single-module vibration-driven system has been greatly improved by Fang and Xu. In [10] and [11], the average velocity of the system was derived through the method of piecewise analysis and the method of averaging. In [10], the possible stick-slip motions are classified into eight types, whose conditions and characteristics were concluded; in [12], from a sliding bifurcation point of view, the stick-slip motion was systematically studied for a vibration-driven system with sinusoidally actuated internal mass. It is shown that the sliding bifurcations organize different types of transitions between slip and sticking motions. In terms of optimization, besides the traditional method, i.e., adjusting the parameters of the internal motion, stick-slip effects was utilized to further increase the average velocity and meanwhile to realize a directed motion. Specially, the friction coefficients and the actuation parameters can be selected in the bifurcation diagram, and the dynamic behaviour of the system can be significantly tailored. The research on vibration-driven system was extended to multi-module systems by Fang and Xu. In [11] and [13], the multi-module systems were built by connecting multiple identical vibration-driven systems together with an elastic element. However, the modules are not driven simultaneously, but with time shifts. Method of averaging was adopted in these multi-module systems to derive the approximate average steady-state velocity. A novel optimization strategy was put forward that by adjusting the actuation phases, the average velocity of the system can be significantly improved. Comparing to the conventional optimization, such optimization method does not call for additional input energy. The stick-slip effects were also studied in a three-module system, where the mechanism for stick-slip motion was analyzed.

On the basis of the vibration-driven with one internal mass, as an improvement, Bolotnik et al. [14,15] added another internal mass to the system. The mass is set to vibrate in the normal direction of the supporting plane so that the dry friction imposed on the system changes periodically. Through optimization, the average velocity of the system is increased by at least 4-fold and direction of the motion of the system can be changed by controlling its harmonic excitations. The theory is experimentally verified by Sobolev and Sorokin in [16].

As can be seen from above, quite a few works have been published to study rectilinear motion of vibration-driven systems with single or more modules, and robots based on some of those theories have been already fabricated out, while studies concerning planar locomotion of vibration-driven system are rare to see. In fact, vibration-driven robots performing planar locomotion can move along many more pathways of different shapes, which greatly increase their applicability in practice. By programming optimal paths for the robots, their movement efficiency can therefore be improved. Besides, studying of planar locomotion also enriches the theory of vibration-driven systems. So it is scientifically significant and practically meaningful to have researches on the planar locomotion of vibration-driven systems.

In the paper, planar locomotion of a vibration-driven system is analyzed. To this end, two movable masses are also set in the system considered. The masses vibrate harmonically in two orthogonal directions: one to propel the system forward, and the other to rotate it. Coulomb dry friction is assumed to act between the system and the horizontal plane. By introducing a nonholonomic constraint, translational and rotary velocities of the system are theoretically obtained. Planar locomotion of the system is analyzed piecewise due to stick-slip effect. As a result, the dynamic system can move along oblique lines with different slopes, which can be controlled by drive frequency and phase difference of internal masses.

2. Mechanical model

2.1. Description of the dynamic system

It is well known that driving an internal mass in a direction can move the body in the same direction. Thus, it may be achievable to control the orientation of the body if another mass is added and is periodically driven in an orthogonal direction. Following this view, we scheme the system as shown in Fig. 1, where the mass of the box-like rigid body is denoted as M, two internal and movable masses as m_1 and m_2, respectively. The designed structure can be also shown in the top view and represented in Fig. 2. The right side of the body is called as “front-end”, the left as “back-end”. In Fig. 2, o_1 is denoted as the centre of body M, $\xi o_1 \eta$ is