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Boundary element method to estimate the time-varying interfacial boundary in horizontal immiscible liquids flow using electrical resistance tomography

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ABSTRACT

Flow of immiscible liquids through horizontal pipe is observed in many industrial process applications. The flow is mainly characterized by its flow regime, relative phase velocities, liquid fraction and water holdup. Electrical resistance tomography (ERT) that provides a cross-sectional image of flow distribution is helpful in monitoring the process. In this paper, the moving interfacial boundary between the immiscible liquids of stratified flow is estimated based on the measured voltages on the pipe surface. Determining the time-varying shape and location of the interfacial boundary has a practical significance. It gives us the cross-sectional liquid fraction which is a key parameter to analyze the hydrodynamic properties of flow. The interfacial boundary shape and location is parameterized with discrete front points and the forward solution is formulated using analytical boundary element method (BEM). The analytic formulation of BEM is simple with less number of unknowns therefore it is computationally efficient. Moreover, BEM discretizes the boundaries alone and is therefore appropriate choice for interfacial boundary estimation. To estimate the time-varying boundary, conventional methods that assume no change in flow distribution within the measurement time of full frame of voltage data are inadequate. Dynamic imaging methods that use one or few sets of data can improve the temporal resolution and hence they are more accurate and practical in realistic situations. Inverse problem is treated as a state estimation problem where the front points are considered as state variables and are estimated using extended Kalman filter. Numerical simulations and phantom experiments are performed to validate the performance of the proposed method.

1. Introduction

Immiscible liquids flow through pipe are observed in many industrial processes [1,2]. For example, transport of oil-water in petroleum industry has immiscible liquids flowing through pipelines. Monitoring the flow characteristics in pipe such as liquid fraction, phase density, relative phase velocity and interface boundary are essential for the design and safety operation of the mechanical equipment [3–5]. Low cost, non-intrusive nature and high temporal characteristics of electrical resistance

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tomography (ERT) makes it a popular imaging modality for process tomography applications. In ERT, voltages are measured due to the applied currents that are injected through the discretely attached electrodes on the surface of the object. Using, the current-voltage relationship, the inner resistivity/conductivity distribution is reconstructed [6]. However, ERT has poor spatial resolution and thus some prior information is often used in image reconstruction. In situation such as immiscible liquids flow, in most cases, there is prior information available about the liquids that are flowing inside the pipe. If the conductivities of these liquids are assumed to be constant and known, then the conductivity reconstruction problem will be transformed to a boundary estimation problem. The shape and location of interfacial boundary can give us information about the liquid fraction through which the relative phase velocities can be determined thus can be helpful in monitoring the transport of fluids through process pipes.

Boundary estimation problems in ERT are classified into two types; closed and open boundary. Closed boundary problem is about recovering the boundaries of inclusions that are enclosed within homogeneous background (for example, bubbles appearing in two-phase flow). In open boundary problem, the flow domain is divided into two disjoint simply connected regions that are separated by an open interfacial boundary (for example, transport of immiscible liquids in a stratified flow). For more information on closed boundary estimation with ERT, please see [7–13]. Several methods have been proposed in literature to represent the shape of an open boundary using known functions [9, 14–17]. With regard to closed boundary, if the interfacial boundary is assumed to be smooth, truncated Fourier series can be used to represent the interfacial boundary. In case, the boundary has a complex shape, higher order Fourier series is required to represent the shape. However, the coefficients of higher order Fourier series are very sensitive to noise and therefore can affect the reconstruction performance. To overcome these problems, Kim et al. [16] introduced an efficient way to represent the open boundary shape. The interfacial boundary is parameterized with discrete front points from the reference points located at the horizon. The front point method is very efficient and can represent any complex shape with fewer unknowns.

The shape reconstruction in ERT is achieved by iterating the forward and inverse problem. In forward problem, the boundary voltages are calculated for a given conductivity distribution and injected currents. The ERT forward problem is often solved using numerical methods as it is difficult to have an analytic solution in general. The analytical solution can be derived for trivial cases [18]. Boundary element method (BEM), finite element method (FEM) and finite difference method (FDM) are generally used for solving the ERT forward problem. FEM is preferred over other methods when there are several heterogeneous regions within the domain. FEM has been previously used for shape estimation problems with ERT [8–17]. In mesh based methods such as FEM the whole domain is discretized with triangular or square elements. Due to FEM discretization, many times there are situations where the anomaly or interface boundary has intersected mesh elements. For the intersected elements on the interfacial boundary, the area weighted resistivity is assigned. In situations with high conductivity contrast of immiscible fluids, such as water and air, even a small error in computation of area weighted resistivities can have a substantial effect on calculated voltages. This can cause the reconstruction performance to deteriorate. Using a very fine mesh or adaptive meshing mechanism can improve the accuracy of FEM solution. But, this leads to increase of computational burden. BEM involves discretizing the boundaries alone and solution is computed on the boundary. Therefore, it is more suited for shape estimation problems [19–23]. In [24], ERT forward problem is formulated with BEM using complete electrode model (CEM) for shape estimation problem. However, this method is computationally intensive as it needs to calculate the potential and fluxes on the interfacial boundary. An analytical computation of boundary integrals is proposed for BEM with CEM to locate anomalies in homogeneous background which is a closed boundary problem [25,26]. In [27,28], BEM is formulated for free surface and stratified flow problem to estimate the open interfacial boundary using ERT. The interfacial boundary is assumed to be stationary i.e. boundary remains unchanged during the time to collect one full frame of independent voltage measurements and the Levenberg–Marquardt method is used as an inverse algorithm for shape estimation. In real situations, the flow is very dynamic and therefore assuming the boundary is static within measurement time of full frame is not practical. Hence, dynamic techniques that consider one or few voltage data instead of full frame are necessary to estimate the time-varying interfacial boundary.

In this paper, a dynamic estimation method is devised to estimate the time-varying interfacial boundary of immiscible liquids in horizontal pipe flows. The forward problem of immiscible liquid flow is formulated using boundary element method with analytic boundary integration. As prior information, it was assumed that the conductivity of liquids is constant during the flow and their value is known thus the conductivity reconstruction problem is transformed to boundary estimation. The interfacial boundary is represented using interpolation of discrete front points. The front points are evaluated at the reference points on the horizon. The flow is considered to be rapidly varying and hence one or few sets of data are used in image reconstruction. The inverse problem is treated as state estimation problem where the front point locations are treated as state variables. The time-varying front point locations are estimated with extended Kalman filter (EKF). Using the proposed BEM with EKF approach, it is possible to estimate the moving interfacial boundary location and shape with good accuracy. Several numerical simulations and phantom experiments with 2D circular domain are performed to show the feasibility of the proposed method.

2. ERT forward problem and boundary parameterization

2.1. Problem description and physical model of ERT

Let us consider a stratified flow inside a circular pipe of radius R with two immiscible liquids (Fig. 1). The flow region Ω is divided into two distinct regions Ω_l and Ω_u whose conductivities are assumed to be known *a priori*. The immiscible liquids in the flow region are separated by an open interfacial boundary ∂D. The interfacial boundary changes rapidly with time due to the