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Development of computational algorithm for multiserver queue with renewal input and synchronous vacation

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ABSTRACT

In this paper, we develop a new computational algorithm for calculating the Markov chain stationary distribution based on a Taylor series approach, where the Taylor series coefficients are expressed in closed-form in terms of the fundamental matrix of the underlying Markov chain. Additionally, we provide an approximate expression for the remainder term of the Taylor series that can be computed in an efficient manner. Specifically, we demonstrate the application of the proposed framework in analyzing a multi-server queueing system with synchronous vacation. The only required assumption of the proposed framework is that the entries of the transition matrix are differentiable functions with respect to a control parameter. Numerical examples are sketched out to illustrate the accuracy of the proposed method.

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1. Introduction

In recent years, there have been significant contributions to the vacation queues. This kind of queues is of great importance, because it has been found to be applicable in the analysis and modeling of computer systems, communication networks, manufacturing and production systems, transportation systems and so forth. Specifically, servers working on supplementary jobs, e.g., performing server maintenance inspection and repairs, or server's failures that interrupt the customer service, are considered to be on vacation from a modeling perspective. Furthermore, proposing various vacation policies provides more flexibility for optimal design of the system. For more detail on this active research area, readers are referred to the excellent surveys on the earlier works of vacation models that have been reported by Doshi [8], Takagi [25,26], Tian and Zhang [28] and Ke et al. [17].

In most studies considered in the literature on queueing systems with vacations, the authors have assumed that there is a single server in the system with an availability of infinite buffer space in front of the server. However, finite buffer queues are more common in several practical applications, and the analysis of finite buffer queue gives a very different system behavior than infinite buffer queue. Besides, multi-server vacation models are more flexible and applicable in practice than single server models. For example, in an airline company having an objective to optimize the utilization of its employees; a specific group of employees has a primary task of load/unload the baggage of passengers. Once they have finished this task, they perform secondary tasks such as aerial ladders or to handle passengers for boarding (i.e., handling passengers at the ticket counter) which is regarded as a vacation. This kind of models is also motivated by situations arising in production systems where a number of identical machines process jobs in order of their arrivals. When there are no jobs, to improve the use of the production facility, the

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machines are switched to do other jobs (they take vacations). Therefore, this production system can be modeled as a finite-buffer multi-server vacation system. In such situations, the synchronous vacation policy could be applied. Other examples of vacation models in various contexts are given in [5].

Existing research works have not considered the finite buffer multiple-server systems with vacations except those related to the Markovian multi-server queues with vacations. In this sense, the GI/M/c/N queueing model with synchronous vacation (servers take vacations together) has not been considered in the literature until now. This paper analyzes the finite capacity GI/M/c queueing model with synchronous vacations. That is, whenever the system is empty, all the servers leave the system for a vacation, and return to the system together when the vacation is completed. If the system is again empty at the moment of the servers return from the vacation, then they leave immediately for another one. Assuming that both the service times and the vacation times are exponentially distributed, the model has been previously analyzed by Chao and Zhao [7] but for infinite buffer queue considering the same vacation policy, and they give a matrix-geometric solution of the arrival-point steady-state queue size probabilities.

The performance analysis of many real systems leads us to consider multi-queue models. The stochastic processes describing the state of such systems are generally very hard to study by analytical methods. This is the reason why computational algorithms are developed for evaluating the performances of such models. The proposed framework involves the use of Taylor series expansion to approximate the entries of the Markov chain stationary distribution vector of the GI/M/c/N queue with synchronous vacation, as polynomial functions of the vacation parameter. Note that if closed-form expressions for the relevant performance measures as a function of the model parameters were readily available, Taylor series expansion would be unnecessary; however, in general, it is difficult to obtain these expressions in closed form. The key thrust of this paper is first to obtain the Taylor series coefficients expressed in closed form as functions of the fundamental matrix [18], and then to apply this new approach to the analysis of the considered queueing model.

The use of Taylor series expansion to study the higher-order sensitivity or the polynomial approximation of stationary characteristics of the underlying Markov chain has been proposed in the literature by several authors (see, e.g., [4,10,11]). In these works, the Taylor series coefficients are expressed in terms of the deviation matrix or the group inverse associated with the underlying Markov chain. In this paper, we provide a new formula for the Taylor series coefficients that typically derived in terms of the fundamental matrix Z, because the accuracy of the computation of Z is of great interest. In this regard, several computational approaches of Z have been proposed in the literature, see for example [3,12–14,19,21,24]. Using this formula, we can obtain a Taylor series for the stationary distribution of the underlying Markov chain. In addition, we demonstrate the application of the proposed framework in analyzing the GI/M/c/N queue with synchronous vacation. We can also note that our framework not only proposes a new approach to the functional approximation of the stationary distribution based on a Taylor series approach, but also provides an approximate expression for the remainder term of the Taylor series that can be computed in an efficient manner.

In this study, we develop a GI/M/c/N queueing model with synchronous vacation. The rest of this paper is organized as follows. Section 2 provides an overview of the related literature on the general input queue with vacations. A sketch of the proposed method is given in Section 3, where we present closed-form expressions for the higher sensitivity of the Markov chain stationary distribution with respect to the control parameter as a function of the fundamental matrix. The GI/M/c/N queueing model with synchronous vacation is analyzed in Section 4. Section 5 presents some numerical examples and the influence of the parameters on the system performance. Section 6 concludes with some possible future research directions.

2. General input vacation models with finite capacity

In this section, we briefly survey the studies on vacation queueing models. We only focus on the models with general inputs and finite capacity.

In recent years, there has been an increasing interest in the analysis of finite buffer queues which are more common in many practical applications. In this direction, Karaesmen and Gupta [15] used the embedded Markov chain to investigate the finite capacity GI/M/1 queueing system with exponentially distributed multiple vacations. The N-policy GI/M/1 queue with finite capacity and exponential vacations has been investigated by Ke [16]. Chao and Rahman [6] analyzed a G/M(n)/1/K queue with state-dependent services and state-dependent vacations. The GI/M/1 queue with limited waiting space and multiple working vacations has been studied by Banik et al. [2]. Sikdar et al. [23] discussed analytical and computational aspects of a finite-buffer batch arrival single-server queue with renewal input and multiple vacations. Later, Yu et al. [29] discussed the $\mathrm{GI}^{[x]}/\mathrm{M}^{b}/1/\mathrm{L}$ queue with multiple working vacations and partial batch rejection. Goswami and Laxmi [9] analyzed the $\mathrm{GI}^{X}/\mathrm{M}/1/\mathrm{N}$ bulk arrival queue with single working vacation policy and partial batch rejection. Recently, Zhang and Hou [30] have studied the GI/M/1/N queue with a variant of multiple working vacation policy.

Most research efforts focused on the single server queues with vacations because the analysis of multi-server vacation models is prohibitively complex. In the case of multi-server vacation models, the servers can either take the same vacation together (called synchronous vacation) or take individual vacations (asynchronous vacations) independently. Chao and Zhao [7] have investigated the multi-server vacation models of both synchronous and asynchronous types and have provided some algorithms for computing the stationary probability distributions and expected performance measures. Moreover, Tian and Zhang [27] have investigated a more general GI/M/c queueing system with phase-type vacations where all servers take multiple vacations together until waiting customers occur at a vacation completion instant.

Throughout this survey, we have also remarked that no study has been done on GI/M/c/N queue with synchronous vacation. Moreover, the majority of the results are expressed in terms of systems of equations of which the solutions are rather

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