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Analytical modeling of eddy current brakes with the application of time varying magnetic fields

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ABSTRACT

Eddy current brakes have a number of potential advantages, i.e. contactless operation, faster response, reduced number of components and easy implementation of various controllers. However, the braking torque generation is limited at low speeds. Here, to increase the braking torque generation, time varying field application is studied. A new analytical model is derived for in-depth theoretical analysis and future controller design purposes. The braking torque generated is calculated using magnetic vector potential and eddy currents. Then, this model was validated using an accurate finite element model. Results show that the braking torque increases with the application of time-varying fields.

1. Introduction

With the advancements in electronics, the "x-by-wire" concept is introduced to the automotive industry by replacing the mechanical components of a device with electrical counterparts in order to improve their performance. Such an improvement was also introduced to the braking technology by means of electromechanical brakes (EMBs).

Eddy current brakes (ECBs) [1–3] are one of the examples of EMBs and they are introduced as potential alternatives to conventional hydraulic friction brakes. Fig. 1 shows a schematic of the ECB concept. When current is applied to the coil (3) on the stationary electromagnet (1), a retarding force will be generated due to the interaction between the magnetic field applied and the eddy currents induced on the surface of the highly conductive rotating disk (2) that is attached to the driving shaft (4).

Braking performance improvements such as faster response time, easy controller implementation, reduced number of components and wiring, contactless operation, silent braking, no material wear due to friction, natural capacity to imitate an anti-lock braking system (ABS) for automotive applications, etc., can be achieved with ECBs. However, limited braking torque generation of current ECBs at low rotational speeds prevents the utilization of ECBs as a stand-alone system in a wide range of practical applications.

The first model of eddy currents generated on a rotating disk was published by Smythe [4]. Smythe defined the relationship between applied external magnetic field and induced fields due to eddy current generation on the surface of a rotating disk using Maxwell–Ampere Law, Faraday's Law and Gauss' Law. His results demonstrated that eddy currents cause demagnetizing fields on electromagnets that oppose the applied field and this results in a variation in the eddy current distribution.

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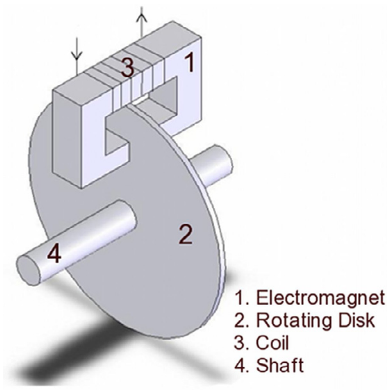

Fig. 1. Eddy current brake (ECB) concept.

Scheiber [5,6] also studied the effects of magnetic field on a moving conductor, however he worked with the assumption of low magnetic Reynolds number, which allows for the effect of the induced fields to be neglected. In [6], using the magnetic potential of the generated eddy currents, Scheiber calculated the braking torque using the total eddy current power input to a rotating conductor.

Wiederick [7] defined a relationship between the eddy currents and the induced voltage due to the applied magnetic field in terms of the electrical resistance of the pole projection area (PPA) that he obtained using empirical data. Heald [8] then revised Wiederick's model using Coulomb's Law to obtain the surface charges and the induced electric field in the PPA.

Studies mentioned above were limited to the low rotational speeds of conductors. Wouterse [9] published his theoretical work on eddy current generation at high rotational speeds. The author's results were comparable to the results of Scheiber and Smythe in the low speed region. Subsequently, Simeu and Georges [10] and Barnes et al. [11] used Wouterse's model in order to design ECBs.

In addition, Burais et al. [12], Peterson [13] and Conraths [14] investigated the eddy current generation on rotating conductors using finite element analysis (FEA). More recently, Lee and Park [15–18] studied the eddy current generation on rotating conductors when DC field is applied and they verified their model using experimental analysis.

While in the above previous studies, it was assumed that time invariant magnetic field (i.e. DC) is externally applied to rotating conductors; Lee and Park [15] proposed and investigated the use of a sinusoidal field (i.e. AC) to increase ECB braking torque generation capacity at low speed region. In [15], the braking torque generation on a 3 PPA ECB configuration was studied using numerical methods. While it was proposed by Lee and Park [15], the explicit use of AC fields to improve an ECB's braking torque at low speeds has been studied and published by our research group in our previous work [19]. In [19], with the help of an accurate finite element model (FEM) of an ECB, the use of alternating magnetic fields with fixed and variable frequencies in various waveforms was investigated at both low and high speeds. The results demonstrated that frequency modulated time varying fields improve the braking torque significantly. In [20], the same model was used to calculate and optimize braking torque generation with eddy current brakes. Unlike the current work presented in this paper, in [20], a multi-PPA ECB configuration was optimized under the application of triangular alternating fields with varying phase angles. This optimization was specifically done for automotive applications and results show the effects of interference between the induced fields at neighboring PPAs.

In order to understand the underlying physics behind eddy current generation by means of time varying (i.e. AC) field application and to design and implement a model-based nonlinear controller for the ECB as a subsequent work, an analytical modeling is carried out in this paper. Eddy currents generated by AC magnetic fields have been previously modeled analytically on various surfaces that are stationary, e.g., in order to study the power loss in electronic components [21,22] or to find defects on conducting sheets in structural health monitoring (SHM) applications [23].

Yannopoulos-Lascaratos and Tegopoulos [24] modeled the eddy currents generated in a stationary cylindrical shell of infinite length when there is alternating magnetic field is present. The authors used the Maxwell's equations and appropriate boundary conditions to solve for the fluxes and the associated loss as well. The authors also obtained numerical solution to the same problem using Gauss–Laguerre method and this model was used to validate the analytical model along with experimental data.

A similar work was published by Sathuvalli and Beyazitoglu [25]. In that work, the Lorentz forces on a conductive sphere placed in alternating (sinusoidal) magnetic field was calculated. The authors expressed the eddy currents generated in terms of the source functions and also accounted for the skin effects. In addition to the analytical model, a numerical model was also obtained and a procedure was given to determine the magnetic pressure distribution on the surface of a liquid metal droplet.

Unlike the above works, Siakavellas [26] studied the eddy current generation on the surface of conductive plates of different shapes using shape dependent factors such as equivalent resistance of the plate and the total circulation current. Using Siakavellas' model, it is possible to estimate the eddy currents generated on the surface of the conductor plate using the shape dependent parameters the author defined; however, the accuracy strongly depends on the shape of the plate. Finally, the author suggested improvements with the consideration of the principle of minimum energy dissipation and Siakavellas validated his analytical model with numerical analysis.