Applied Mathematical Modelling 40 (2016) 1193–1198

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

Uncertain master–slave synchronization with implicit minimum saturation level

Giuseppe Fedele [a,*], Andrea Ferrise [a], Francesco Chiaravalloti [b]

[a] Department of Informatics, Modeling, Electronics and Systems Engineering, University of Calabria, Via Pietro Bucci, 42C, Rende 87036, Italy
[b] Department of Physics, University of Calabria, Via Pietro Bucci, Rende 87036, Italy

ARTICLE INFO

Article history:
Received 19 August 2013
Revised 20 January 2015
Accepted 22 July 2015
Available online 4 August 2015

Keywords:
Chaos synchronization
State feedback control
Lyapunov theory

ABSTRACT

In this note, the exponential synchronization of a class of chaotic uncertain systems is addressed. A linear state feedback control is used which guarantees the synchronization when uncertainties are present both in linear and nonlinear parts of the slave system dynamic. It is shown that, with some a-priori rough bounds on the slave uncertainties, the closed loop stability can be guaranteed. Numerical simulations are presented to show the effectiveness of the proposed method.

1. Introduction

Chaotic dynamical systems exhibit sensitive dependence on the initial conditions and parameters values. Despite this, since the pioneering works of Pecora and Carroll [1,2], it has become known that it is possible to force two chaotic systems to synchronize using a drive-response configuration. The basic schema of the process of driven synchronization provides a master system, which evolves freely, and a slave system which is forced, via a coupling signal, to exponentially follow the dynamics of the master. Due to its potential useful applications, e.g. in cryptography and secure communications [3–7], or in modeling chemical reactions [8–10] and process in biological systems [11–14], the phenomenon of chaos synchronization has attracted widespread attentions from many fields, and various methods for chaos control have been developed such as those based on optimal control [15], PID control [16], linear state feedback control [17], linear matrix inequality (LMI) [18], just to name a few. Most of the schemes to achieve chaos synchronization were originally designed for identical chaotic systems in which it is assumed that systems parameters are completely known in advance. However, a more general case is represented by systems with some or all parameters possibly unknown or perturbed. As a consequence, several approaches have been recently proposed to deal with such an issue [19–21]. In this paper a Lyapunov-based approach is discussed to force an uncertain slave system following the dynamic of an equivalent master system. Lyapunov techniques have been widely applied to solve the problem under investigation, see [22] and references therein. The novelty of the proposed approach consists in the synthesis of the forcing signals according to known bounds on the slave system uncertainties. As shown in the sequel, the method requires some less-restrictive constraints for the dynamics of the master and slave systems satisfied by a wide class of chaotic systems, such as Lorentz, Chen and Chua. The paper is organized as follows: in Section 2 the problem formulation is presented together with the required assumptions upon the systems dynamics. The synchronization method is reported in Section 3. Section 4 contains a numerical simulation showing the effectiveness of the approach applied to master–slave Chua's systems synchronization. The last section is devoted to conclusions.

* Corresponding author. Tel.: +39 0984 494720; fax: +39 0984 494713.
E-mail addresses: gfedele@dimes.unical.it (G. Fedele), aferrise@dimes.unical.it (A. Ferrise), francesco.chiaravalloti@unical.it (F. Chiaravalloti).

http://dx.doi.org/10.1016/j.apm.2015.07.010
S0307-904X(15)00433-3/

2. Model description and assumptions

A chaos-based communication system usually consists of two chaotic systems at the transmitter and receiver ends, which are called the master system and the slave system. At the transmitter end, the master system is

$$\dot{x}(t) = \phi(x(t)), \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the vector of state variables and $\phi(x) : \mathbb{R}^n \to \mathbb{R}^n$ is a continuous function that takes into account both the linear and the nonlinear parts of the system. Several groups of chaotic systems such as Lorentz, Chen, Lu and Chua's systems can be represented in the form (1).

At the receiver end, the slave system is written in the following form:

$$\dot{y}(t) = \phi(y(t)) + \Delta_\phi(y) + u(t), \tag{2}$$

where the nonlinear function $\Delta_\phi(y) : \mathbb{R}^n \to \mathbb{R}^n$ takes into account both uncertainties on linear and nonlinear part of the master system. The controller $u(t) \in \mathbb{R}^n$ has been added to the slave system in order to synchronize its states $y(t)$ with the states of the master system $x(t)$.

Assumption 1. The slave system is characterized by a norm-bounded perturbation vector Δ_ϕ, i.e.

$$\left\| \Delta_\phi(\cdot) \right\| \leq \gamma_s$$

with γ_s a positive known bound.

Defining the synchronization error as $e(t) \triangleq y(t) - x(t)$ and using (1) and (2), the synchronization error dynamics is determined as follows:

$$\dot{e}(t) = \phi(x(t) + e(t)) - \phi(x(t)) + \Delta_\phi(y(t)) + u(t). \tag{3}$$

The main aim is to find a control signal $u(t) \in \mathbb{R}^n$ that makes states of the slave system to follow the states of the master systems exponentially.

Assumption 2. The nonlinear transition function $\phi(\cdot)$ satisfies the following Lipschitz condition

$$\left\| \phi(y(t)) - \phi(x(t)) \right\| \leq L \left\| y(t) - x(t) \right\| \tag{4}$$

with positive known Lipschitz constant L.

Similar assumptions have been largely adopted in the devoted control literature. In general, as remarked in [23–25], Assumption (4) is not restrictive since the trajectories of chaotic systems are always bounded.

3. Synchronization criteria

In this section some sufficient conditions are derived to ensure exponential synchronization between the master and the slave system, based on Lyapunov stability theory.

Theorem 1. *If the forced signal of the slave system is chosen as*

$$u(t) = -(\lambda + L)e(t) - \gamma_s \frac{e(t)}{\|e(t)\|} \tag{5}$$

with design parameter $\lambda > 0$, *then the error between* $y(t)$ *and* $x(t)$ *converges exponentially to zero and hence the slave system can achieve exponential synchronization with the master system.*

Proof. Consider the following Lyapunov candidate function:

$$V(t) = \frac{1}{2} e(t)^T e(t). \tag{6}$$

The time derivative of $V(t)$ is

$$\begin{aligned} \dot{V}(t) &= e(t)^T \left[\phi(x(t) + e(t)) - \phi(x(t)) + \Delta_\phi(y(t)) + u(t) \right] \\ &\leq L\|e(t)\|^2 + \gamma_s \|e(t)\| + e(t)^T u(t). \end{aligned} \tag{7}$$

By considering the input signal in Eq. (5), the following relationship can be obtained

$$\dot{V}(t) \leq -\lambda ||e(t)||^2 \equiv -2\lambda V(t)$$

from which the proof follows. Moreover

$$V(t) \leq e^{-2\lambda t} V(0) \tag{8}$$

and, as a consequence, the error $e(t)$ is monotonically decreasing. □