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Elastic bending and buckling of a steel composite beam with corrugated main core and sandwich faces—Theoretical study

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A B S T R A C T

The subject of the paper is a seven-layer steel beam with transverse sinusoidal corrugated main core and two sandwich faces with steel foam cores. Two analytical models of the beam are formulated. The first model includes only a shear effect of two cores of the faces, while the second one includes a shear effect of the main core and two cores of the faces. The first model is described by two differential equations of equilibrium, and the second one by three differential equations of equilibrium. These equations are solved for three-point bending and axial compression of the beam. Moreover, the FEM model is elaborated and deflections and critical forces are calculated for the example beams. Results of these three models are presented in Figures.

1. Introduction

The sandwich structure is widely used in many branches of engineering, especially aircraft, naval and automotive industries. The bases of the theory of sandwich structures were formulated by Libove and Butdorf, and Reissner in the mid of 20th century. Analytical models of sandwich structures are described in many monographs, for example: Libove and Hubka [12], Plantema [20], Volmir [25], Allen [1]. The theory and applications of sandwich structures intensively developed in 20th century are a subject of contemporary studies. Carlsson et al. [3] derived tension, shear, bending and twisting rigidities for sandwich structures with corrugated core. Buannic et al. [2] derived the equivalent properties of sandwich plate with corrugated core. Carrera [4] described a historical review of Zig-Zag theories for multi-layered structures. Cheng et al. [5] proposed the finite element method to derive an expression for the equivalent stiffness of sandwich structures with various types of cores. Magnucka-Blandzi and Magnucki [14] discussed strength and stability problems of sandwich beams and its effective design. Peng et al [19] used element-free Galerkin (EFG) method for static analysis of stiffened plates based on first-order shear deformation theory (FSDT). Isaksson et al. [6] analyzed elastic properties of corrugated board panel. Kazemahvazi et al. [10] and [11] modeled and tested experimentally the failure mechanisms of corrugated cores of sandwich structures. Seong et al. [21] studied metallic sandwich plates with bi-directionally corrugated cores. Magnucka-Blandzi [13] presented mathematical modeling of a rectangular sandwich plate with metal foam core. Smith et al. [22] and [23] discussed mechanical properties, experimental characterization and manufacturing of steel foams for structural components. Jasion et al. [7] analyzed global and local buckling of sandwich plates with metal foam cores. Jasion et al. [8] studied global and local buckling of sandwich plates with metal foam cores. Szyniszewski et al. [24]

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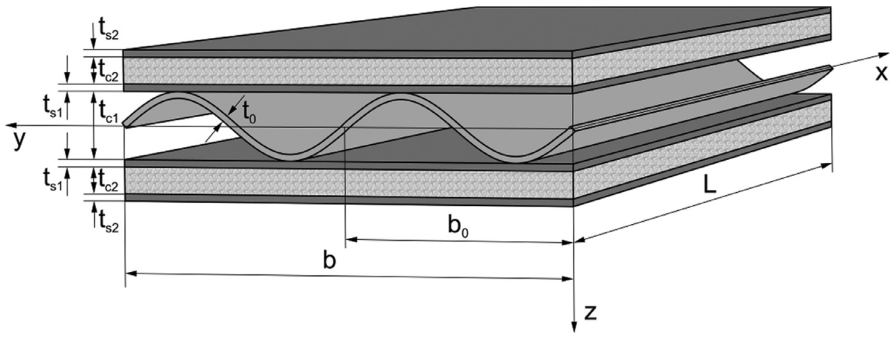

Fig. 1. The seven-layer thin-walled beam.

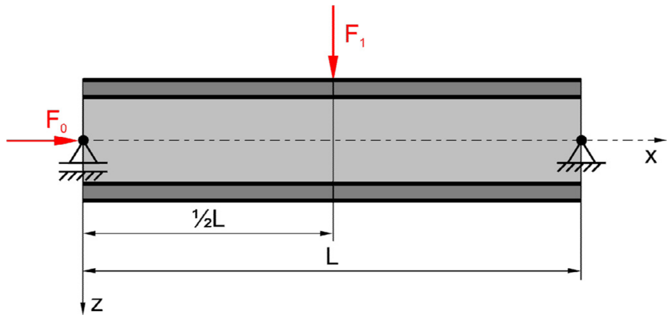

Fig. 2. The load cases of the simply supported beam.

described local buckling strength of steel foam sandwich panels. Magnucki et al. [15] discussed effective design of rectangular plate with corrugated core. Magnucki et al. [16] and [18] investigated bending and buckling problems of orthotropic sandwich beams with three-layer faces. Magnucki et al. [17] presented the comparison of results of the numerical-analytical analysis and experimental investigation of sandwich beams with corrugated cores. Jing et al. [9] investigated numerically dynamic response of cylindrical sandwich shells with aluminum foam cores subjected to air blast loading.

The subject of the study is the steel composite beam with transverse sinusoidally corrugated main core and three-layer faces (Fig. 1). The composite beam of the length L and the width b consists of seven thin-walled layers with three cores and four flat sheets. The main core is a corrugated layer in the middle of thickness t_{c1}. The corrugation pitch of the main core is b_0. The corrugation depth is $t_{c1} - t_0$, where t_0 is thickness of the corrugated sheet. Two other cores are internal layers of faces of thickness t_{c2}. The flat sheets are of thickness t_{s1} – inner sheets, and t_{s2} – outer sheets. Materials of the main core and sheets are isotropic. The outer cores are made of the metal foam.

The simply supported sandwich beam of the length L carries concentrated transverse force F_1 or a compressive axial force F_0. The first configuration of force and supports is known as three-point bending, the second one – axial compression (Fig. 2).

2. Analytical investigation

The hypothesis of deformation of a flat cross section of the beam is a basis for the formulation of displacements, strains, stresses and equilibrium equations.

2.1. The first model of the beam

The shearing effect in the core of upper and lower faces is considered. In the main corrugated core the shearing effect is neglected – classical (Euler–Bernoulli) beam theory is used.

The field of displacements and strains with consideration of the first hypothesis (Fig. 3) take the following form:

- the outer upper sheet $-(1/2 + x_1 + x_2 + x_3) \le \zeta \le -(1/2 + x_1 + x_2)$

$$u(x,\zeta) = -t_{c1}\left(\zeta\frac{dw}{dx} + x_2\psi(x)\right), \quad \varepsilon_x^{(u-s2)} = -t_{c1}\left(\zeta\frac{d^2w}{dx^2} + x_2\frac{d\psi}{dx}\right), \quad \gamma_{xz}^{(u-s2)} = 0, \tag{1}$$