Applied Mathematical Modelling 40 (2016) 1287–1299

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

Free vibrations of thin rectangular nano-plates using wave propagation approach

M.R. Ilkhani [a,*], A. Bahrami [b], S.H. Hosseini-Hashemi [a,c]

[a] Impact Research Laboratory, Department of Mechanical Engineering, Iran University of Science and Technology, Tehran, Iran
[b] Department of Mechanical Engineering, Eastern Mediterranean University, TRNC Mersin 10, Magosa, Turkey
[c] Center of Excellence in Railway Transportation, Iran University of Science and Technology, Narmak, 16842-13114 Tehran, Iran

ARTICLE INFO

Article history:
Received 3 January 2014
Revised 6 May 2015
Accepted 25 June 2015
Available online 3 September 2015

Keywords:
Rectangular nano-plate
Propagation matrix
Reflection matrix
Vibration analysis
Graphene sheet

ABSTRACT

The wave propagation approach is one of the most powerful techniques used in vibration problems. This method gives the propagation and reflection matrix which is useful for the analysis of mechanical energy transmission in micro/nano devices like energy harvester devices. In this paper, wave propagation approach is used to analyze the free vibrations analysis of thin rectangular macro and nano-plates. Firstly, the propagation and reflection matrices for thin rectangular plates are derived. Then, these matrices are combined to provide a concise and exact approach for obtaining the natural frequencies of the thin rectangular plates. In the macro scale, solution obtained by this approach is exactly the same as those derived by classical method. In Nano-scale, the results are compared with Molecular dynamic simulation of a graphene sheet. Finally, the behavior of reflection coefficients, as an index for energy transmission, under changing the frequency for various nonlocal parameters is studied.

1. Introduction

In classical method, the boundary conditions are applied to the general solution of the differential equation of motion to obtain the natural frequencies. An alternative method is to describe vibrations as propagating waves travelling in the structure. This method known as wave propagation technique is a simple, non-iterative and efficient method for calculating the frequency of the structures. Unlike other methods, the wave propagation technique provides analytical expressions for transmitted, reflected power and energy flow of the waves in the waveguides using the derived reflection and transmission matrix coefficients. As a result, the wave propagation method is useful not only for vibration analysis but also for energy transmission analysis. Wave propagation, transmission and reflection in solids have been studied by a number of researchers. The majority of researches used this method to analyze the vibration of beams. Mace [1] considered the wave propagation approach to study the free vibrations of Euler Bernoulli beams and Timoshenko beams [2]. Tan [3] presented the wave motions in an axially strained, rotating Timoshenko shaft. Also, the wave propagation in a rotating Timoshenko shaft was considered by Argento and Scott [4]. Lee et al. [5] used wave approach to analyze the non-uniform waveguides such as non-uniform bars and non-uniform Euler Bernoulli beams whose properties vary rapidly but deterministically. Bahrami and Loghmani [6] used the wave approach to analyze the non-uniform rod with exponential cross-section. Mei et al. [7] presented the wave approach for free and forced vibrations of axially loaded cracked Timoshenko beams. Lee et al. [8] applied the wave method in thin, uniform, and curved

* Corresponding author. Tel.: +98 77240540; fax: +9877240540.
E-mail address: ilkhani@iust.ac.ir (M.R. Ilkhani).

http://dx.doi.org/10.1016/j.apm.2015.06.032
S0307-904X(15)00444-8/

beams with constant curvature to obtain the natural frequencies of the curved beams. In 2010, an exact wave-based analytical solution was presented by Mei [9] for obtaining the natural frequencies of classical planar frame structures, in which the coupling effect between bending and longitudinal vibrations was taken into account. In 2011, Mei [10] used a wave vibration approach to study the effects of lumped end mass on bending vibrations of a Timoshenko beam. Furthermore, Mei [11] applied the wave approach to obtain the natural frequencies and mode shapes of single-story multi-bay planar frame structures. Recently, Mei [12] analyzed the vibration of single-story multi-bay planar frame structures using the wave approach, in which the effects of rotary inertia, shear deformation, and joint model on vibration characteristics were taken into account. Bahrami et al. [13] used a modified wave approach to analyze a non-uniform Euler beam. Also, the wave propagation technique is used for analysis of cylindrical shells. Zhang et al. [14] calculated the frequencies of thin cylindrical shells using wave approach based on Love's shell theory. Zhang et al. used this approach for coupled vibration of fluid-filled shells [15], submerged shells [16] and cross-ply laminated composite shells [17]. Xuebin [18] investigated the free vibration of a circular cylindrical shell using wave propagation approach based on Flugge theory. Most of the mentioned references used the wave propagation approach in one dimensional waveguides or planar frame structures, in which the waves propagate along one dimensional structural element and are reflected and transmitted at joints and boundaries. To the best knowledge of the authors, there has been only one attempt to apply the wave approach for two dimensional uniform waveguide structures. Recently, Bahrami et al. [19] used the wave approach for uniform circular membranes and sectorial membranes to obtain the natural frequencies of these structures. There have been no attempts to apply the wave approach for analyzing other two dimensional waveguide structures such as plates.

In recent years, the great potential of nano-structures has attracted researchers' attention. They concentrate on the prediction of physical behavior of them to improve the performance of nano-scale devices. The graphene sheet is the main body in the nano-structures area because all the carbon nanotubes (CNTs) and fullerene are formed graphene sheets. Also, they are used as resonators and sensors in so many electromechanical devices. So, analyzing their behavior helps engineers to improve their systems performance. The main computational tools used for analysis of the nano structures are categorized into three groups: molecular mechanics, hybrid atomistic-continuum mechanics and continuum mechanics. Among these groups, the continuum mechanics is a reliable, simple and computationally non-expensive method compared to the other two.

The main assumption in the classical continuum mechanics is that the stress at a point is a function of strain at that point. But, in the nano-scale, the spaces in the molecular lattices are comparable with the dimensions of such structures. So, the continuum theories need some appropriate changes to consider the size effects. In the 90's, some efforts were made to consider this effect on continuum model by Mindlin and Eshel [20], Green and Rivlin [21] and Mindlin [22,23] which because of their multiplicity of unknowns were so complex. In recent years, Eringen [24] considered the size effect and proposed a new nonlocal elasticity theory. In the nonlocal elasticity, stress at a point is a function of strain on all point of the body. So many researchers analyze the mechanical behavior of nano-structures, especially nanotubes using this theory. Various analytical and semi-analytical methods like generalized differential quadrature (GDQ), finite element and Molecular simulation beside Euler or Timoshenko nonlocal beam theory are used to analyze the free vibration of single or double wall CNTs [25–32]. Also, wave propagation and dispersion in the CNTs has been studied by some researchers. Wang et al. [33–35] studied the longitudinal and flexural wave propagation in the CNTs. Narendar and Gopalakrishnan [36] studied nonlocal scale effect on the wave propagation in multi-walled CNTs. Wang et al. [37] analyzed wave propagation in the CNT under axial load and elastic matrix.

Dynamic behavior of graphene sheets have been investigated by so many researchers, too. The free vibration of single layer and double layers nano-plates have been studied by Pradhan and his co-workers [38–41]. They used nonlocal thin plate theory and molecular dynamics simulator for their analysis. Ansari el al. [42] used MD simulation and presented a scale factor for free vibration analysis of graphene sheets in nonlocal Mindlin plate theory. Arash and Wang [43] studied free vibration of single and double layer graphene sheets using nonlocal thin plate theory and generalized differential quadrature method. Ansari et al. [44] investigated vibration characteristics of embedded multi-layered graphene sheets with nonlocal elasticity. An excellent review on the application of nonlocal theories in the nano-scale structures were made by Arash and Wang which can be used as a good reference for any research [45]. Also, Hashemi et al. [46] studied free vibrations of nano-plates using first order shear deformation nonlocal plate theory and an exact approach. The behavior of nano-plates under longitudinal wave propagation was investigated by Wang et al. [47]. Also, the effects of nonlocal scale on the sound wave dispersion of monolayer graphene sheets were studied by Narendar et al. [48]. It should be mentioned that the wave propagation phenomenon which is considered by so many researchers in literature for both macro scale and nano-scale, have a big difference with the wave propagation technique. The wave propagation technique is a method which uses propagation and reflection matrices to find the behavior of the bodies. But, wave propagation analysis is based on the investigation of a body response to a specific type of incident wave. So, according to a beneficial literature review and to the best of the authors' knowledge, there is no literature for analysis of the free vibration of macro/nano plates using wave propagation technique. In the present paper, an attempt is made to present another method to analyze free vibration of macro/nano plate using wave approach for Levy type combinations of boundary conditions. The propagation and reflection matrices presented will be helpful for future work correlated with wave power transmissions or reflections in the nano-plates as the power harvester devices. The natural frequencies obtained by this method are compared with the molecular dynamic simulation results in the literature and a nonlocal scale factor is presented for graphene sheet with better accuracy compared to other researches. A benchmark results are presented for any combination of classical boundary conditions in a practical range of scale factors. Finally, the effect of changing