Applied Mathematical Modelling 40 (2016) 1300–1314

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

Propagation of Torsional surface waves in an inhomogeneous anisotropic fluid saturated porous layered half space under initial stress with varying properties

Sushant Shekhar[a,b,*], Imtiyaz A Parvez[a,b]

[a] Academy of Scientific and Innovative Research, New Delhi-110 001, India
[b] CSIR-Fourth Paradigm Institute, Bangalore-560037, India

ARTICLE INFO

Article history:
Received 16 October 2013
Revised 10 October 2014
Accepted 1 July 2015
Available online 3 September 2015

Keywords:
Torsional surface waves
Inhomogeneity
Porous media
Initial stress
Whittaker's function
Modified Bessel function

ABSTRACT

We study the behavior of torsional surface waves when they propagate through inhomogeneous fluid saturated porous layer over a homogeneous porous half space. The layer has three types of inhomogeneity, viz; linear, quadratic and exponential, varies with depth as rigidity, density and initial stress. We assume both media under compressive initial stresses and the analysis is based on the Biot's theory. The effect of inhomogeneity of the layer in the propagation of torsional surface waves have been studied. The dispersion equations are derived for each case and solved by an iterative method (Newton–Raphson method). It is observed from the numerical calculation that the presence of initial stress and inhomogeneity of the medium affect significantly to the phase velocity of torsional surface waves. Also, propagation of torsional surface waves depend upon the medium in which they propagate as torsional surface waves propagate fastly in presence of elastic half space in comparison to porous half space.

1. Introduction

The dynamical behavior of fluid saturated porous media has great importance in many fields, such as seismology, earthquake engineering, soil dynamics and fluid dynamics. Because of the importance in these fields, the study of torsional surface waves has considerable attention for researchers in recent years. Many of the implications and applications of this medium in different research fields were studied by the researchers. Notable among these are [1–10].

Most of the theoretical seismologists and earth scientists are interested for investigating the seismic waves in layered media. Surface wave propagation over homogeneous and inhomogeneous medium is a well known and prominent feature of the wave theory. An amount of information about the propagation of seismic waves is available in [11,12]. Although there are lot of literature available for the propagation of surface waves, such as the Rayleigh, Love and Stoneley waves but very less information is available for torsional surface waves. These waves are horizontally polarized but give a twist to the medium when they propagate. In seismogram, some disturbances find in between the arrival of Rayleigh and Love waves. Earlier, sufficient informations were not available for these disturbances, were termed as 'noise' and are ignored in the study of seismic waves. These 'noise' may be observed in the form of torsional surface waves that propagate in the non-homogeneous earth.

* Corresponding author at: CSIR-Fourth Paradigm Institute, Solid Earth Modelling group, NAL belur campus, wind tunnel road, Bangalore, Karnataka 560037, India. Tel.: +918025051346; fax: +918025220392.

E-mail address: sushant.shekhar85@gmail.com, sushant@csir4pi.in (S. Shekhar).

http://dx.doi.org/10.1016/j.apm.2015.07.015
S0307-904X(15)00442-4/

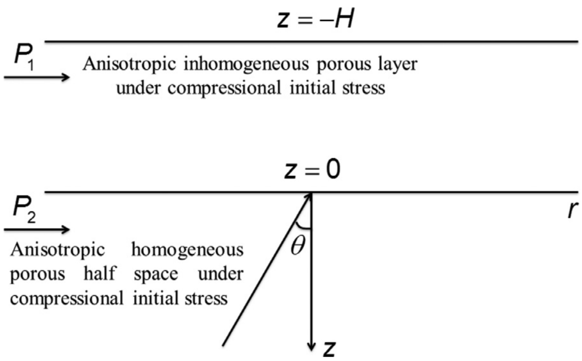

Fig. 1. Geometry of the problem.

Few studies also discussed the various aspect of mathematical theory of elasticity by [13]. Information related to sound waves in porous media is found from the books by [14,15]. Theory of propagation of elastic waves in statistically isotropic fluid saturated porous solid has been first studied by [16–19]. Information related to application of mathematics in seismology is available in the book by [20] and [21]. Quite a good amount of information related to propagation of seismic waves in different earth layers are contained in the book by [22]. Study of wave motions in fluid-saturated porous rocks has been done by [23]. [24] studied torsional surface waves in inhomogeneous elastic media. By using Biot's theory, [25–27] in a series of papers, studied the propagation of torsional surface waves in different medium under different condition. [28] studied the propagation of plane waves in transversely isotropic dissipative half space. Propagation of Love waves in a transversely isotropic fluid saturated porous medium have been studied by [29]. [30] shows that torsional surface waves exist in a homogeneous gradient elastic half-space. Using Biot's theory, [31–35] in a series of papers, studied the propagation of torsional surface waves in different medium, different type of irregularities and under different condition.

The methodology, we are using is similar to that given in [6,7]. In their study, they assumed that both the media are transversely isotropic fluid saturated porous solids, the upper layer is inhomogeneous and half-space is homogeneous, because, the upper crust is more inhomogeneous as compared to the lower crust. But for numerical calculation, they assumed that the half space as elastic. In the present study, we are considering the same geometry with an attempt to understand the behavior of torsional surface waves in inhomogeneous anisotropic fluid saturated porous layer over homogeneous anisotropic fluid saturated porous half space under compressive initial stresses. For numerical calculation, we discuss two cases for half space: (1) elastic half space ([29]), (2) fluid saturated porous half space ([23]). The comparison of these two cases have been shown graphically for torsional surface waves.

2. Formulation of problem

We are considering the propagation of torsional surface waves in an inhomogeneous anisotropic fluid saturated porous layer of thickness H under compressive initial stress P_1 along radial direction over a homogeneous anisotropic fluid saturated porous half space under compressive initial stress P_2 along same direction as shown in Fig. 1.

The coordinate system is taken to be cylindrical with $z-$axis pointing vertically downward as shown in Fig. 1. The interface is located at $z=0$, and the radial direction (r-axis) is taken in the direction of wave propagation. The dynamical equation of an initially stressed poro-elastic medium can be obtained by Biot's dynamical equations (neglecting the viscosity of the liquid and body forces) for porous layer under compressional initial stressed as given by [16–19]. Here, we are using indices $l=1$ for inhomogeneous porous layer and $l=2$ for homogeneous porous half space.

$$\begin{aligned}
\frac{\partial s_{rr}^{(l)}}{\partial r}+\frac{1}{r}\frac{\partial s_{r\theta}^{(l)}}{\partial \theta}+\frac{\partial s_{rz}^{(l)}}{\partial z}+\frac{s_{rr}^{(l)}-s_{\theta\theta}^{(l)}}{r}-P_i\left(\frac{\partial \omega_z^{(l)}}{\partial \theta}+\frac{\partial \omega_\theta^{(l)}}{\partial z}\right) &= \frac{\partial^2}{\partial t^2}\left(\rho_{rr}^{(l)}u_r^{(l)}+\rho_{r\theta}^{(l)}U_r^{(l)}\right),\\
\frac{\partial s_{r\theta}^{(l)}}{\partial r}+\frac{2}{r}s_{r\theta}^{(l)}+\frac{1}{r}\frac{\partial s_{\theta\theta}^{(l)}}{\partial \theta}+\frac{\partial s_{\theta z}^{(l)}}{\partial z}-P_i\frac{\partial \omega_z^{(l)}}{\partial r} &= \frac{\partial^2}{\partial t^2}\left(\rho_{rr}^{(l)}u_\theta^{(l)}+\rho_{r\theta}^{(l)}U_\theta^{(l)}\right),\\
\frac{\partial s_{rz}^{(l)}}{\partial r}+\frac{1}{r}\frac{\partial s_{z\theta}^{(l)}}{\partial \theta}+\frac{\partial s_{zz}^{(l)}}{\partial z}+\frac{1}{r}s_{rz}^{(l)}+P_i\frac{\partial \omega_z^{(l)}}{\partial \theta} &= \frac{\partial^2}{\partial t^2}\left(\rho_{rr}^{(l)}u_z^{(l)}+\rho_{r\theta}^{(l)}U_z^{(l)}\right)
\end{aligned} \tag{1}$$

$$\begin{aligned}
\frac{\partial s^{(l)}}{\partial r} &= \frac{\partial^2}{\partial t^2}\left(\rho_{r\theta}^{(l)}u_r^{(l)}+\rho_{\theta\theta}^{(l)}U_r^{(l)}\right),\\
\frac{\partial s^{(l)}}{\partial \theta} &= \frac{\partial^2}{\partial t^2}\left(\rho_{r\theta}^{(l)}u_\theta^{(l)}+\rho_{\theta\theta}^{(l)}U_\theta^{(l)}\right),
\end{aligned}$$