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Semi-layerwise analysis of laminated plates with nonsingular delamination—The theorem of autocontinuity

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ABSTRACT

The proposed semi-layerwise approach captures the mechanical behavior of delaminated composite plates using four equivalent single layers independently of the lay-up. Two equivalent single layers are applied for both the top and bottom parts of a delaminated plate. The updated version of the system of exact kinematic conditions formulates the continuity of the in-plane displacements between the neighboring layers, the location of the global reference plane of the plate and – as important additions compared to previous papers – the continuity of shear strains, their derivatives and curvatures, respectively. The method is demonstrated using the first-, second- and third-order plate theories. As examples, simply supported delaminated plates are considered.The continuity between the delaminated and undelaminated plate regions is established through the theorem of autocontinuity. The J-integral is calculated along the straight delamination front and compared to the results of the virtual crack closure technique. The results indicate that the first- and third-order plate theories provide the best solutions, and give good approximation even in those cases when the previous models failed, i.e., when the delamination is asymmetrically placed between two layers and it is close to the free surface of the plate.

1. Introduction

Composite materials are susceptible to many type of damage modes [1–3]. One of them is delamination fracture induced by manufacturing and installation defects [4–8], low velocity impact [9–13], free edge effect [14,15] and the usage of notches and indentations [16,17] for the installation of the structure. The presence of cracks and delaminations in laminated composite beams and plates reduce significantly the stiffness and strength [18,19], moreover alter significantly the dynamical properties of the structure [20–25]. The stress intensity factor (SIF) [26–30] and energy release rate (ERR) [31–33] are the basic parameters of linear elastic fracture mechanics for the material characterization against delamination fracture (onset and propagation). The mode-I [34–43], mode-II [44–49], mode-III [50–63], and mixed-mode I/II [42,44,64–76], I/III [77,78], II/III [54,79–89], and I/II/III [90–92], fracture in composite materials is characterized by standard and nonstandard beam and plate specimens. While for beams the analytical solutions are relatively easy to develop, for plates similar exact solutions are difficult to obtain.

The classical (CLPT) or Kirchhoff [93–96], first-order (FSDT) [97–101], second-order (SSDT) [102,103,103–106], general third-order (TSDT) [107–111] and Reddy's [112–115] third-order shear deformation plate theories are widely-used in the composite

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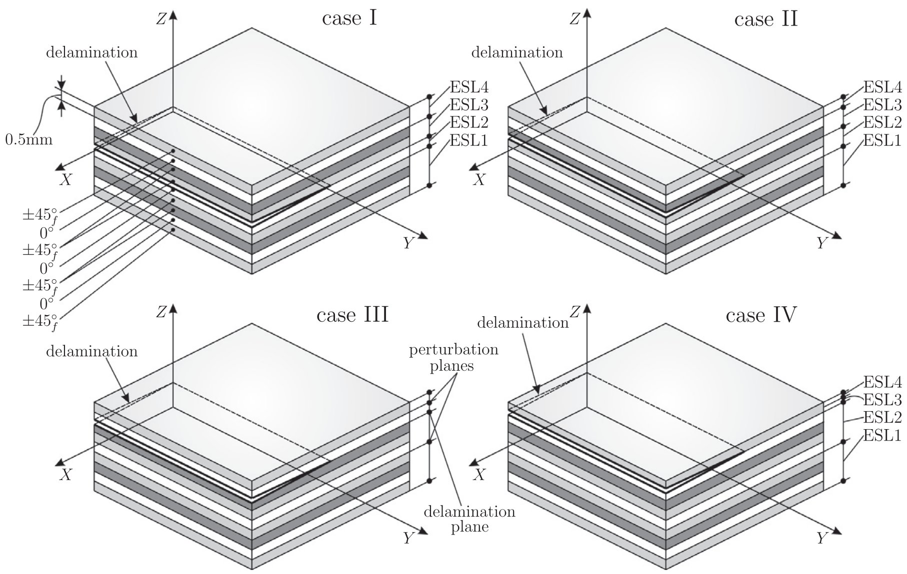

Fig. 1. Plate elements with orthotropic plies and the position of the delamination over the thickness of the plate.

literature to solve different plate problems in the field of statics, dynamics and stability. The layerwise (or zig-zag) theories [93,116–120] make it possible to calculate the interlaminar stresses more accurately than by using the ESL methods. However, the plate theories are applied only partially in the analysis of fracture mechanical tests, in most of the cases a 3D finite element (FE) analysis is preferred [121,122]. The finite element modeling of delaminations and cracks in 3D structures is computationally expensive. Typical examples in this respect are the different fracture mechanical configurations to determine the mode-III energy release rate [78,122].

Recently many articles were published on the modeling of the bending of composite plates with delamination using CLPT, FSDT, SSDT, TSDT for symmetrically [123–125] and asymmetrically [106,114,125] delaminated plates and by using layerwise approximations for in-plane loading and cylindrical bending [126–129]. The method of two ESLs was introduced in the former works and it was shown that for symmetric (midplane) delamination each method works well; however Reddy's third-order theory provides the best accuracy compared to FE results if the delamination is asymmetrically placed between two adjacent layers [114]. The mentioned problems involve typically mixed-mode II/III fracture conditions, in case it has already been shown that the coupling between the mode-II and mode-III SIFs could be significant [130,131].

This paper proposes the method of four ESLs for the delamination modeling of laminated composite plates. Three different theories are applied: FSDT, SSDT and general TSDT. The main aspect of the formulation is that the delamination plane divides the plate into a top and a bottom subplate. These subplates are modeled by two ESLs. The kinematic continuity is provided by the updated version of the system of exact kinematic conditions (SEKC) [114,125], the novelty is the specification of the continuity conditions with respect to the shear strain derivative and curvature. The displacement field satisfying the continuity conditions are formulated and the governing equations are derived based on variational calculus. Two simply-supported plates with delamination are examined and the Lévy plate formulation is used to reduce the system of PDEs to system of ODEs. The continuity between the delaminated and undelaminated portions has been formulated and it was highlighted that the number of constants in the solution functions is less than the number of continuity conditions. Therefore, in conjunction with the proposed 4ESL method the theorem of autocontinuity is introduced and a proof is given, as well. This theorem makes it possible to ensure the continuity between the delaminated and undelaminated portions by assigning the so-called autocontinuity parameters, which ensures the automatic continuity of first-, second- and third-order displacement terms in a reduced form. The displacement and stress fields in the laminated plates were determined by FSDT, SSDT and TSDT and at certain sections located in the delamination front were plotted along the thickness of the plate. The distributions of the J-integrals and mode mixities along the delamination front were also calculated and were compared to the results of the virtual crack closure technique (VCCT). The agreements and disagreements of the 2D analytical results with the numerical models are discussed.

2. Semi-layerwise laminated plate theory – the method of four ESLs

The concept of the semi-layerwise modeling is shown in Fig. 1. The plate elements contain an interfacial delamination, which divides the plate into a top and a bottom layer. Each layer is divided into further two ESLs. In other words the whole laminate is modeled by four ESLs – two above and below the delamination plane. The interface plane between two adjacent ESLs is the perturbation plane. The ESLs can be modeled by different plate theories. In this work the FSDT, SSDT and TSDT are applied to capture the mechanical fields. The components of the displacement field in general third-order plates can be written