Applied Mathematical Modelling 40 (2016) 1372–1386

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

Extension of MULTIMOORA method with interval numbers: An application in materials selection

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ARTICLE INFO

Article history:
Received 15 February 2015
Revised 23 May 2015
Accepted 1 July 2015
Available online 4 September 2015

Keywords:
Multiple attribute decision making (MADM)
MULTIMOORA
Interval numbers
Materials selection

ABSTRACT

The MULTIMOORA method is developed using interval numbers based on fuzzy logic concept and a novel preference technique to deal with complex uncertain problems. By utilizing the fuzzy concept and considering relative weights for attributes, assessment values are obtained for subordinate parts of the interval weighted MULTIMOORA. We suggest an interval comparison matrix to calculate aggregate preference as an index to find the interval maximal objective reference point vector and rank the assessment values of the interval weighted ratio system and the full multiplicative form. The theory of dominance is employed to consolidate the subordinate ranks into the final ranking. Because of the reasonable assumption of alternative ratings as interval numbers in industrial problems and straightforward mathematics of the interval weighted MULTIMOORA, the proposed method can be effectively utilized by decision makers. A case study regarding materials selection of power gears is solved using the proposed method. Our final ranking for the practical case is verified with two interval-based and several crisp MADM methods.

1. Introduction

Decision-making is the process of specifying the best alternative from all feasible options. The category of MADM methods is a useful set of tools to solve the problems of selection from a limited number of alternatives considering multiple beneficial and non-beneficial attributes. An MADM technique determines how attributes information must be assessed to reach an appropriate selection. Given the decision matrix and a decision-making approach, the decision maker has to choose the best alternative and/or rank the complete set of alternatives.

In traditional MADM methods, the ratings and attributes weights are assumed to be crisp without uncertainty about the values. However, responses of alternatives to attributes and relative importance of attributes are not necessarily exact in the real world. Moreover, decision makers' judgment is often vague and they may not present their preferences with exact numerical values. To deal with uncertain elements of a decision problem, fuzzy and stochastic approaches are widely employed. In fuzzy decision making, the parameters are presumed to have given membership functions. In stochastic decision making, the parameters are assumed to have specified probability distributions. In reality, a decision maker may not be able to determine the membership function or probability distribution in an uncertain environment. Therefore, application of interval numbers tends to facilitate the decision making process at least in some cases.

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http://dx.doi.org/10.1016/j.apm.2015.07.019

The multi-objective optimization on the basis of ratio analysis (MOORA) and its updated form MULTIMOORA methods belong to the category of newly developed MADM approaches. As an effective and simple MADM technique, the MULTIMOORA method facilitates the selection process. In this paper, we extend the MULTIMOORA method utilizing interval numbers based on fuzzy logic theory and a novel comparison technique.

The remainder of the present paper is organized as follows. A related literature and explanation of the research gap are provided in Section 2. The original MULTIMOORA method is presented in Section 3. Description of interval numbers including the developed comparison technique for intervals is given in Section 4. In Section 5, the proposed method is discussed. Section 6 discusses an application of the proposed method. Conclusions are finally made in Section 7.

2. Literature review

2.1. Survey of interval MADM methods

Many MADM methods have been developed using interval numbers. Jahanshahloo et al. [1] suggested an extension of the technique for order preference by similarity to ideal solution (TOPSIS) to deal with decision making problems with interval numbers. Sayadi et al. [2] extended the compromise ranking also known as vlse kriterijumska optimizacija kompromisno resenje (VIKOR) technique considering interval data. Pan et al. [3] utilized the linear additive utility function and composite utility variance to formulate an interval MADM approach. Fa-Dong et al. [4] introduced a unique interval MADM method with loss aversion. Cao and Wu [5] recommended an extension of continuous ordered weighted geometric (COWG) operators to solve multiple attributive group decision making (MAGDM) problems with interval numbers.

2.2. Survey of development, extensions, and applications of the MULTIMOORA method

Brauers and Zavadskas [6] introduced a new MADM method called the MOORA. The MOORA technique and its developed form that is called MULTIMOORA method [7] have been widely employed in different applications. Brauers et al. [8] extended the crisp MULTIMOORA method employing the fuzzy numbers. Liu et al. [9] determined the risk of failure modes utilizing the extended MULTIMOORA method with fuzzy numbers. Mandal and Sarkar [10] employed the fuzzy MOORA methodology for selection of an optimal intelligent manufacturing system.

2.3. Survey of applications of MADM methods in materials selection

Because of large number of materials and variety of manufacturing techniques, materials selection may become a complicated task for an engineer or designer. A wide range of MADM approaches has been employed in materials selection by earlier researchers, such as MOORA [11], TOPSIS [12], VIKOR [13], and diverse versions of elimination and choice expressing the reality (ELECTRE) also recognized as outranking method [14].

2.4. Research gap

Baležentis and Zeng [15] utilized interval-valued fuzzy numbers for derivation of an extension of the MULTIMOORA method. Interval linguistic variables were employed to develop the MULTIMOORA technique by Liu et al. [16]. Stanujkic et al. [17] used gray numbers to generate another form of the MULTIMOORA method. An extension of the ratio system, i.e., the first part of the MOORA method, based on interval data was suggested by Stanujkic et al. [18]. To the best of authors' knowledge, no study has considered interval data for ratings of alternatives in formulation of all subordinate parts of the MULTIMOORA method nor utilized the dominance theory to reach the final ranking. To fill the research gap, we develop a unique extension of the MULTIMOORA method with interval alternatives ratings and a novel interval comparison matrix to rank alternatives. We employ the proposed technique to select the optimal alternative in a practical materials selection problem.

3. The MULTIMOORA method

The MOORA method introduced by Brauers and Zavadskas [6] consists of two parts, namely the ratio system and the reference point approach. Brauers and Zavadskas [19] extended the MOORA method considering the full multiplicative form. The updated technique, named MULTIMOORA, incudes the MOORA subordinate parts and the full multiplicative form. The MULTIMOORA method starts with a decision matrix $\mathbf{X}$ in which x_{ij} shows the response of ith alternative to jth attribute, $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$:

$$\mathbf{X} = [x_{ij}]_{m\times n}. \tag{1}$$

To obtain comparable and dimensionless ratings, the decision matrix is normalized:

$$\mathbf{X}^* = [x^*_{ij}]_{m\times n}, \tag{2}$$

in which x^*_{ij} is the normalized response of ith alternative regarding jth attribute. Normalization is a comparison between each response of an alternative to an attribute, as a numerator, and a denominator that is a representative for all alternatives responses