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Biased reduction method by combining improved modified pole clustering and improved Pade approximations

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ABSTRACT

This paper presents a mixed method to reduce the order of the linear high order dynamic systems by combining improved modified pole clustering technique and improved Pade approximations. The denominator of the reduced order model is computed by improved modified pole clustering while the numerator coefficients are obtained by improved Pade approximations. The proposed method is competent in generating '*k*' number of reduced order models from the original high order system. The superiority of the proposed method is concluded through numerical examples taken from literature and compared with existing well- known order reduction methods.

1. Introduction

Model order reduction (MOR) technique is a smart idea in computer aided design (CAD) area since few decades. It converts an original high order system into low order system, yet still retains the main characteristics of original system in an optimum manner. Therefore, by converting the original model to a reduced model, the higher order original system can be analyzed easily. Model order reduction technique has become an interesting tool in the field of engineering design such as control theory, power system, fluid dynamics etc. For linear dynamic systems, there are several reduction techniques in time domains and frequency domains both, are available in the literature [1–4]. Also, some mixed reduction techniques have been suggested by combining two frequency domain methods [5–7]. In clustering technique [8], zeroes and poles are freely collected to form clusters and then clusters are formulated by inverse distance measure (IDM) criterion to find cluster centers. For the reduced order model, in method [9], to synthesize the reduced order denominator polynomial, the pole cluster centers are obtained by pole clusters. Further, Vishwakarma [10] has suggested a seven step algorithm based on IDM criterion known as modified pole clustering technique, which is capable of generating more dominant cluster centers as compared to obtained in [8,9]. In the proposed method, a small modification in the algorithm [10] is suggested in order to improve the pole cluster centers and calling it improved modified pole clustering technique. Pade approximation method [11], which is computationally simple and retains even small initial time-moments, is used to match the forced response of the original system. The drawback of the method [11] is its inability to retain stability in the reduced order model even though the original system is stable. This problem was removed by improved Pade approximations method [12]. The important features of the both techniques i.e. improved modified pole clustering

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technique and improved Pade approximations [12] are combined together to develop a more powerful algorithm for order reduction of the linear dynamic systems.

The proposed biased method has few major advantages i.e. computational simplicity, stability retention and robustness etc. In addition to these advantages, it is able to generate 'k' number of reduced order models for $k^{th}-$order reduction, and also a reduced order model with better response may be further preferred for the design and analysis. Sometimes, reduced order models obtained by proposed method have tendency to turn into non-minimum phase, which may be considered as a drawback of the method.

2. Statement of the problem

Let the $n^{th}-$order original system is expressed as

$$G(s) = \frac{N(s)}{D(s)} = \frac{a_0 + a_1 s + a_2 s^2 + \ldots\ldots + a_{n-1}s^{n-1}}{b_0 + b_1 s + b_2 s^2 + \ldots\ldots b_n s^n} \tag{1}$$

Where $(a_0, a_1, a_2 \ldots a_{n-1})$ and $(b_0, b_1, b_2 \ldots b_n)$ are known scalar quantities.

Let the reduced order model having order '\$k\$' is expressed as

$$R_k(s) = \frac{N_k(s)}{D_k(s)} = \frac{c_0 + c_1 s + c_2 s^2 + \ldots\ldots + c_{k-1}s^{k-1}}{d_0 + d_1 s + d_2 s^2 + \ldots\ldots d_k s^k} \tag{2}$$

Where $(c_0, c_1, c_2 \ldots c_{k-1})$ and $(d_0, d_1, d_2 \ldots d_k)$ are unknown scalar quantities.

The problem is to find the reduced order model (2) from the original $n^{th}-$order system (1) such that it retains the time and frequency response specifications of the original system as much possible.

3. Description of the method

The proposed order reduction method for synthesizing the $k^{th}-$order reduced model is described through the following subsections.

3.1. Determination of the reduced order denominator polynomial

The following points must be considered while making the $k^{th}-$order reduced model.

- Make individual clusters for real and complex poles.
- The pole clusters of left half s-plane should not have a single pole of right half s-plane.
- Poles on the imaginary axis have to be retained in the reduced order model.
- Poles on the origin must be retained in the reduced order model.

Consider the $i^{th}-$pole cluster with '\$r\$' number of real poles such that $|p_1| < |p_2| \ldots\ldots < |p_r|$ and its pole cluster center is represented as p_{ei}. Similarly, consider the $j^{th}-$pole cluster with '\$m\$' number of complex poles i.e. $[(\alpha_1 \pm j\beta_1), (\alpha_2 \pm j\beta_2), \ldots\ldots (\alpha_m \pm j\beta_m)]$, such that $|\alpha_1| < |\alpha_2| \ldots\ldots < |\alpha_m|$ and its complex pole cluster center is

$$\varphi_{ej} = A_{ej} \pm jb_{ej}, \text{ where } \overset{*}{\Phi}_{ej} = A_{ej} + jb_{ej} \text{ and } \overset{\bullet}{\Phi}_{ej} = A_{ej} - jb_{ej}.$$

For finding the real and complex pole cluster centers, an iterative algorithm is shown in Section 3.1.1, which is the improved version of modified pole clustering [10].

3.1.1. An iterative algorithm for improved modified pole clustering

An iterative powerful algorithm [10,13] with small modification in step-6 for finding improved modified pole cluster center consists of the following seven steps.

Step-1: Let a cluster have '\$r\$' real poles i.e. $|p_1| < |p_2| \ldots < |p_r|$.

Step-2: Set $j = 1$.

Step-3: Estimate pole cluster center

$$q_j = \left[\sum_{i=0}^{r} \left(\frac{-1}{|p_i|} \right) \div r \right]^{-1}.$$

Step-4: Set $j = j + 1$.

Step-5: Generate an improved modified pole cluster center from

$$q_j = \left[\left(\frac{-1}{|p_1|} + \frac{-1}{|q_{j-1}|} \right) \div 2 \right]^{-1}.$$