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# Multi-fidelity shape optimization of hydraulic turbine runner blades using a multi-objective mesh adaptive direct search algorithm

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ABSTRACT

A robust multi-fidelity design optimization methodology has been developed to integrate advantages of high- and low-fidelity analyses, aiming to help designers reach more efficient turbine runners within reasonable computational time and cost. An inexpensive low-fidelity inviscid flow solver handles most of the computational burden by providing data to the optimizer by evaluating objective functions and constraint values in the low-fidelity phase. An open-source derivative-free optimizer, NOMAD, explores the search space, using the multi-objective mesh adaptive direct search optimization algorithm. A versatile filtering algorithm is in charge of connecting low- and high-fidelity phases by selecting among all feasible solutions a few promising solutions which are transferred to the high-fidelity phase. In the high-fidelity phase, a viscous flow solver is used outside the optimization loop to accurately evaluate filtered candidates. High-fidelity analyses results are used to recalibrate the low-fidelity optimization problem. The developed methodology has demonstrated its ability to efficiently redesign a Francis turbine blade for new operating conditions.



## 1. Introduction

From an energy production perspective, the moving component of a hydraulic turbine, the runner, plays a key role in its operation. Designing a runner currently relies extensively on the designer's intuition and experience. Although runner designers employ CFD tools to evaluate their designs, there is a strong need to integrate more tightly CFD analyses to obtain more automatic and efficient design processes. A full range of CFD methods has been utilized in the optimization of hydraulic turbine runner blades; from low-fidelity inviscid models (e.g. using potential flow, Holmes and McNabb [1]) to high-fidelity viscous models (e.g. using a turbulent RANS solver, Franco-Nava et al. [2]). However none of these methods can, by itself, entirely fulfill industrial design needs. On one hand, low-fidelity CFD simulations are not accurate enough in their prediction of flow behavior, mainly due to shortcomings in the physics. On the other hand, high-fidelity CFD analyses cannot be used in the main optimization loop, since they are too expensive and slow for iterative industrial blade design processes. Surrogate-based optimization approaches have been employed, whereby computationally inexpensive models are used in lieu of high-fidelity models. These approaches may be divided into functional and physics-based surrogates. Although functional surrogates have been used for blade shape

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## Nomenclature

| | |
|---|---|
| $B_L$ | Lower bound of design variables |
| $B_U$ | Upper bound of design variables |
| C | Design characteristic |
| $C_\mu$ | Turbulent kinetic energy constant |
| $C_{\varepsilon 1}$, $C_{\varepsilon 2}$ | Turbulent model constants |
| $C^*$ | Targeted design characteristic |
| G | Sieving grid size |
| g | Gravity |
| H | Height or head |
| $I_{Obj}$ | Indices used for objectives |
| $J_{Cons}$ | Indices used for constraints |
| k | Turbulent kinetic energy |
| $k_L$ | Relaxation factor of characteristic limit correction |
| $k_{OP}$ | Relaxation factor of operating condition correction |
| $k_s$ | Shrinkage factor |
| $k_T$ | Relaxation factor of design characteristic correction |
| $\vec{n}$ | Wall normal vector |
| OP | Operating point |
| $\widetilde{OP}$ | Operating point of minimum characteristic |
| P | Pressure |
| R | Cluster radius |
| $R^N$ | N-dimensional Euclidean space |
| $\vec{r}$ | Radial coordinate vector |
| $S_{in}$ | Inlet swirl |
| U | Characteristic limit |
| $U_i$, $U_j$, $U_k$ | Cartesian mean velocity vectors |
| $\vec{V}$ | Velocity vector |
| $\vec{W}$ | Relative velocity vector |
| X | State variable |
| x | Cartesian position |
| Y | A set of geometric design variables |
| $Y^*$ | A set of geometries |
| y | Independent design variable |
| Z | Mapped geometry |
| $Z^*$ | A set of mapped geometries |
| z | Vertical coordinate in cylindrical system |

*Greek symbols*

| | |
|---|---|
| $\rho$ | Density |
| $\varepsilon$ | Turbulent dissipation rate |
| $\Omega$ | Angular velocity |
| Ø | Potential function |
| $\theta$ | Angle |
| $\mu$ | viscosity |
| $\mu_t$ | Turbulent viscosity |
| $\sigma_\varepsilon$, $\sigma_k$ | Turbulent model constants |

*Subscripts*

| | |
|---|---|
| B | Selected band of feasible solutions |
| bc | Boundary condition |
| C | Candidate |
| Cons | Constraint |
| c | Cluster |
| F | Feasible |
| i,j,k,l,m,q,t | Counting indices |
| N | Number of design variables |