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Simultaneous determination of the strain hardening exponent, the shear modulus and the elastic stress limit in an inverse problem

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ABSTRACT

This paper is devoted to simultaneous determination of the strain hardening exponent, the shear modulus and the elastic stress limit in an inverse problem. The inverse problem consists of determining the unknown coefficient $f = f(T^2), T^2 := |\nabla u|^2$ in the nonlinear equation $u_t - \nabla.\big(f(T^2)\nabla u\big) = 2t,\ (x, y, t) \in\ \Omega_{\mathcal{T}} := \Omega \times (0, \mathcal{T}),\ \Omega \subset \mathbb{R}^2$, by measured output data (or additional data) given in the integral form. After we solve direct problem using a semi-implicit finite difference scheme, a numerical method based on discretization of the minimization problem, steepest descent method and least squares method is proposed for the solution of the inverse problem. We use Tikhonov regularization to overcome the ill-posedness of the inverse problem. Numerical examples with noise free and noisy data illustrate applicability and accuracy of the proposed method to some extent.

1. Introduction

According to the deformation theory of plasticity, stress–strain relation between deviators is described by the Hencky correlation

$$\sigma_{ij}^D = 2g(\Gamma^2)\epsilon_{ij}^D, \quad i, j = 1, 2, 3.$$

Then the following relation holds between the intensities of shift strain $\Gamma := \left(2\epsilon_{ij}^D\epsilon_{ij}^D\right)^{\frac{1}{2}}$ and tangential stress $T := \left(\frac{1}{2}\sigma_{ij}^D\sigma_{ij}^D\right)^{\frac{1}{2}}$

$$T = g\big(\Gamma^2\big)\Gamma, \tag{1.1}$$

where the function $g(\Gamma^2)$ describes the elastoplastic properties of the material and is sometimes called the modulus of plasticity. Eq. (1.1) can be formally regarded as a general condition encompassing different phases strain. Thus, putting $g\big(\Gamma^2\big) = \frac{\tau_s}{\Gamma}$, we obtain the Von Mises's criterion $T = \tau_s$; while putting $g\big(\Gamma^2\big) = G$, we obtain the case of Hooke's elastic medium, where $T = G\Gamma$ and $G = E/(2(1+\nu))$ is the modulus of rigidity (shear modulus), $E > 0$ is the Young's modulus, $\nu \in (0, \frac{1}{2})$ is the Poisson coefficient. The shear modulus is defined as the ratio of shear stress to the shear strain. It describes

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Nomenclature

g	modulus of plasticity
G	modulus of rigidity (shear modulus)
E	Young's modulus
ν	Poisson coefficient
$\mathbb{F}$	class of admissible coefficients
Ω	cross section of a bar
$\partial\Omega$	boundary of Ω
φ	angle of twist per unit length
$T^2 := \|\nabla u\|^2$	stress intensity
$u(x, y)$	Prandtl stress function
${T_0}^2 := \max_{x \in \Omega}\|\nabla u\|^2$	elastic stress limit
M	theoretical value of the torque
$\mathcal{M}$	measured value of the torque
$\mathcal{T}$	final time
$\langle ., .\rangle$	inner product
$\|\|.\|\|_\infty$	maximum norm
$\|\|.\|\|_2$	L_2 norm in Ω
$L_2(\Omega)$	set of square integrable functions on Ω
$J(f)$	cost functional
τ	time step
w_h	uniform mesh
h_1	mesh step in x direction
h_2	mesh step in y direction
N_1	number of mesh points in x direction
N_2	number of mesh points in y direction
N	number of measurements
u	exact solution
u_h	approximate solution
u_0	initial approximation
εu_h	absolute error
$O(.)$	Landau's symbol
δu_h	relative error
κ	strain hardening exponent
$\mathbf{T}$	transpose of a matrix
∇	gradient
λ	regularization parameter
ϵ	stopping criterion
q	number of points in t direction
h	differential step for κ
k	differential step for G
m	differential step for ${T_0}^2$

an object's tendency to shear when acted upon by opposing forces. Also it is used to determine how elastic or bendable materials evolve if they are sheared, which is being pushed parallel from opposite sides. The Poisson coefficient for some materials such as aluminum, bronze, copper, ice, magnesium, molybdenum, monel metal, nickel silver are 0.334, 0.34, 0355, 0.33, 0.35, 0.307, 0.315, 0.322 respectively. Since the Poisson coefficient of the aforementioned materials are around 0.3, it is assumed to be 0.3 throughout this paper. We note that changing of this value affect numerical results but does not affect the applicability and efficiency of the method given in Section 3.

According to the deformation theory of plasticity, the function $g(\Gamma^2)$ satisfies the following conditions [1]:

$$\begin{cases} c_1 \le g(\Gamma^2) \le c_2, \\ g(\Gamma^2) + 2g'(\Gamma^2)\Gamma^2 \ge c_3 > 0, \forall \Gamma^2 \in \left[{\Gamma_*}^2, {\Gamma^*}^2\right], \\ g'(\Gamma^2) \le 0, \\ \exists {\Gamma_0}^2 \in ({\Gamma_*}^2, {\Gamma^*}^2) : g(\Gamma^2) = G, \ \forall \Gamma^2 \in \left[{\Gamma_0}^2, {\Gamma^*}^2\right], \end{cases} \tag{1.2}$$