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Flow field and traverse times for fan forced injection of fumigant via circular or annular inlet into stored grain

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ABSTRACT

Fan forced injection of phosphine gas fumigant into stored grain is a common method to treat infestation by insects. For low injection velocities the transport of fumigant can be modelled as Darcy flow in a porous medium where the gas pressure satisfies Laplace's equation. Using this approach, a closed form series solution is derived for the pressure, velocity and streamlines in a cylindrically stored grain bed with either a circular or annular inlet, from which traverse times are numerically computed. A leading order closed form expression for the traverse time is also obtained and found to be reasonable for inlet configurations close to the central axis of the grain storage. Results are interpreted for the case of a representative 6 m high farm wheat store, where the time to advect the phosphine to almost the entire grain bed is found to be approximately one hour.

1. Introduction

Infestation of stored grain by insects can cause contamination, odors, molds, and heat damage that reduces the market value of the grain [1] or, if left untreated, makes the grain unsaleable to most buyers [2]. A common method used for eliminating the insect populations in grain storage is fumigation by phosphine, which is cost effective, easy to use, and also a residue-free treatment [2,3]. However, for successful insect control, the phosphine must be effectively distributed to all areas of the grain bed and kept in contact with the insects for sufficient time at the required concentration. If not, there will be potential zones that can provide areas of refuge where insects can survive and breed.

In view of the above, a good understanding of how the phosphine is distributed in stored grain is important. This can be facilitated by understanding both the flow patterns and the traverse time, defined as the time taken for phosphine entering from the inlet to reach a specified position in the grain bulk. The latter was introduced in early work on grain storage [4] and subsequently employed in a number of studies as a useful interpretive tool (e.g. [5,6]. In terms of the relevant physics that requires modelling, phosphine gas is driven mainly by advection during fan forced fumigation. While molecular diffusion is involved the contribution is comparatively small. For example, for stored grain with grain bed height 6 m, typical seepage velocities are of the order 10^{-2} m/s [7] and the diffusion coefficient is of the order 10^{-5} m^2/s [8], yielding a Peclet number

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of the order 10^3. Hence, with such an application in mind, fan forced gas flow through the stored grain by advection only is considered here.

The overall objective of this study is to progress further the recently initiated work by the authors [9] to investigate fumigation in stored grain. Here, analytic solutions are the main focus. Apart from providing insights into typically simplified forms of more realistic problems, they can also provide accurate solutions against which numerical simulation models involving more complex physical processes or geometrical grain storage configurations can be validated. The specific aim of the present study is to derive closed form analytic solutions for pressure, velocity and streamlines in a cylindrical grain storage with an annular inlet. The approach also permits the investigation of a circular inlet as a special case. Consistent with earlier studies, incompressible Darcy flow through a porous medium is considered, applicable to relatively low phosphine injection velocity at the inlet.

Regarding earlier work, a number of analytical solutions for two dimensional planar gas flow or traverse times related to grain storage under similar modelling assumptions have been obtained [5,6,10–13], although none investigate the geometrical configurations considered in this study. To the authors' knowledge, the most relevant prior work to the present one is a closed form solution for the traverse time in a circular cylindrical store open to the atmosphere having a conical base and centrally positioned inlet [5]. However, the expression for the traverse time solution so obtained was an approximation only, and the inlet was treated as a point source. Regarding some other planar two dimensional flows, analytic solutions for calculating the pressure drop within grain storage was obtained by Hunter [12] using a conformal mapping approach. Formulae for traverse times were later developed by Hunter [5] for a few particular geometries, including some of the plane flows in his earlier work [12]. Incompressible Darcy flow in a rectangular bin open to the atmosphere studied by Hunter [5] was further studied in detail by Goudie et al. [11]. The gas entering the grain bed was also treated as a point source inlet at the centre of the bin floor. They reproduced the analytic solution in [5] for pressure, velocity, streamlines, and traverse time in the limit of infinite grain bed height, and also analysed the finite height case. It was found that their limiting solution for semi-infinite height was sufficiently accurate for grain beds higher than 1.25 m. The flow was also extended to investigate Ergun's equation by using perturbation expansions. An analytic solution to this Ergun flow problem was later obtained by de Ville and Smith [10], but by simplifying from two dimensional to unidirectional flow. A perturbation analysis of Ergun flow was also applied to a triangular domain [13] to infer various bounds for the flow solutions and approximate corrections to model Ergun rather than Darcy flow. Conformal mapping along with matched asymptotic expansions was used by Smith and Jayas [6] to obtain approximate solutions for incompressible Darcy flow in grain contained by a rectangular symmetrical bin geometry. However, in contrast to [5] and [11], the inlet where the gas entered the bin was treated as a finite curved shape instead of a point source and approximate flow solutions were obtained. The main focus was to understand the conditions under which the traverse time could be used to understand the heat and mass transfer processes that were considered in the study.

2. Model equations

The equation of motion used here to model phosphine flow driven by advection in grain storage is Darcy's law [14]. Using tildes over variables to denote dimensional quantities, Darcy's law is:

$$\tilde{\boldsymbol{v}} = -\frac{k}{\mu}\nabla\tilde{p}, \tag{1}$$

which relates the pressure $\tilde{p}$ and superficial velocity of the gas $\tilde{\boldsymbol{v}}$, defined as the volume of gas crossing a unit area of porous medium per unit time. This equation is accurate for low velocity flows [6] as considered here. A number of other modelling assumptions are also implied in the above formulation. For example, effects of gravity are neglected as the specific gravity of phosphine is similar to air [15]. We also assume negligible temperature variation in the grain bed during fumigation [16] and a uniform pore distribution with height. The permeability k and dynamic viscosity μ are taken as experimentally determined constants, which for the case of pure phosphine in wheat grain are $k = 5.78 \times 10^{-9}\,\mathrm{m}^2$[17], and $\mu = 1.1 \times 10^{-5}\,\mathrm{kgm}^{-1}\mathrm{s}^{-1}$. Further, we assume incompressible flow [7,18], so (1) is solved together with the continuity equation, $\nabla \cdot \tilde{\boldsymbol{v}} = 0$. In this case, the pressure satisfies Laplace's equation $\nabla^2\tilde{p} = 0$, which expressed in cylindrical coordinates $(\tilde{r}, \tilde{z})$ is:

$$\frac{\partial^2\tilde{p}}{\partial\tilde{r}^2} + \frac{1}{\tilde{r}}\frac{\partial\tilde{p}}{\partial\tilde{r}} + \frac{\partial^2\tilde{p}}{\partial\tilde{z}^2} = 0. \tag{2}$$

The flow is calculated for an unsealed circular cylindrical grain store fumigated with phosphine from a single inlet attached to the base, as illustrated in Fig. 1. The gas is pumped continuously into the grain bed at an assumed low and constant mass flow rate Q which for modelling purposes is described via a uniform velocity v_0 m/s across an inlet of size $\tilde{b} = \tilde{f} - \tilde{d}$, where $\tilde{d}$ and $\tilde{f}$ are the radial distances from the central symmetry line to the annular inlet's inner and outer radius, respectively. The case $\tilde{d} = 0$ corresponds to a circular inlet pipe. While in practice, the inlet is sometimes positioned at locations such as the vertical wall, for the purpose of this study the inlet location is restricted to the base to allow an axisymmetric analysis.

The grain surface $\tilde{z} = \tilde{h}$ is considered open to the atmosphere and the condition $\tilde{p} = \tilde{p_a}$ is used, where $\tilde{p_a}$ is the atmospheric pressure. The confining silo vertical wall and base of the grain bed are assumed impermeable, except at the inlet.