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Reduced-order modeling of linear time invariant systems using big bang big crunch optimization and time moment matching method

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ABSTRACT

In this paper, a new approach is proposed to approximate the high-order linear time invariant (LTI) system into its low-order model. The proposed approach is a mixed method of model order reduction scheme consisting of recently developed big bang big crunch optimization algorithm and the time-moment matching method. This proposed method is applicable to single-input single-output, multi-input multi-output system, and time delayed LTI systems. The proposed approach is substantiated with various numerical examples of low and high-order systems. The results show that the reduced-order models preserve both transient and steady state conditions of original systems. Further, the results are also compared with the existing approaches of reduced order modeling which show exceptional improvement in integral square error (*ISE*) and other time domain specifications.

1. Introduction

The concept of model order reduction (MOR) is basically practiced in the field of systems and control engineering where the properties of dynamical systems are analyzed to reduce their complexity and retain their inputoutput behavior as much as possible. MOR simplifies the understanding of the system, and minimizes the computational burden in the simulation studies. Moreover unlike the design of complex H_∞ and μ synthesis based control schemes, it enables the control practitioners to design simple control laws thereby making controller computationally and cost efficient [1]. Advanced robust control concepts such as H_∞ control and μ synthesis are widely used in various engineering applications wherein the design schemes produce high-order controllers even for a simple second-order plant [2]. To cope with the aforementioned challenges faced when dealing with large-scale dynamical systems, various MOR approaches have been proposed using variety of concepts. Over thirty years of research in MOR for LTI system, the developed methods conceptualize features such as dominant pole retention, singular value decomposition and Hankel norm based approximation, singular perturbation approximation, Krylov subspace method, H_∞-optimization methods [3–6]. One of the important trends in these studies is the optimization (minimization) of integral error criterion between the actual plant and its reduced model.

In recent years, there is a widespread interest and research in *evolutionary optimization techniques* mainly because of its intuitiveness, ease of implementation, and the ability to effectively solve highly non-linear, mixed integer optimization problems of complex engineering systems. Now-a-days, these evolutionary optimization techniques unified with conventional

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MOR techniques have shown highly promising results. Among various evolutionary techniques, particle swarm optimization and genetic algorithm are considered to be highly reliable algorithm for solving optimization problems [7]. On the same lines, a novel universe inspired evolutionary technique so called *Big Bang Big Crunch* (BBBC) can be used for order reduction of systems of various complexities [8]. Recently, a mixed method is proposed for order reduction of LTI systems for both single-input single-output (SISO) and multi-input multi-output (MIMO) systems [9]. This method is based on unification of BBBC and Routh approximation method. It is observed that this method is better than the existing method and has been applied for low-order systems but a more accurate and improved results can also be achieved. Therefore, in this paper, an alternative mixed method is proposed to obtain reduced model using BBBC and time moment matching method. In this approach, numerator coefficients of reduced-order transfer function model are optimized using BBBC technique and denominator is evaluated using the time moment matching method from full-order plant's information. Here, this proposed method is applied to SISO, MIMO and time delayed system. The comparison of the existing approaches and proposed approach has been carried which show the remarkable improvement in integral square error (*ISE*) and other performance parameters.

The organization of this paper is as follows: Section 2 briefly summarizes the preliminaries of the concepts involved in MOR, formulation of the proposed MOR scheme is presented in Section 3, simulation studies of the proposed method has been conducted on different types of systems in Section 4, and the conclusions are drawn in Section 5.

2. Background materials

In this section, we present the background material of the concepts that are involved in the proposed scheme.

2.1. Model reduction from control system's perspective

In control theory, MOR can be defined as follows.

Definition 1. Let $G(s)$: $u \to y$ be the transfer function of the full-order system with order n, then the model reduction scheme seeks a reduced-order transfer function model $\tilde{G}(s) : u \to \tilde{y}$ with order $\tilde{n}$ so that $\tilde{n} < n$ and for the same input $u(t)$, we have $\tilde{y}(t) \approx y(t)$.

Remark 1. For the sake of definiteness, the type of systems considered in this paper is proper LTI systems. The proposed method is limited to transfer function matrices such that

(1) $G(s)$ are rational and stable, i.e., the poles lie on the left-half of the s-plane.
(2) In frequency domain, $G(j\omega)$ is nonzero for all ω including $\omega = \infty$.

In other words, MOR leads to optimization problem in the following manner.

Definition 2. MOR defines the problem of finding the reduced-model $\tilde{G}(s)$ from the full-order plant $G(s)$ using optimization formulation such that

$$\text{minimize } ISE = \int [y(t) - \tilde{y}(t)]^2 dt$$
$$\text{subject to } \; ISE < \varepsilon$$

where ε is an error tolerance.

Remark 2. We have selected *ISE* criterion because it quickly eliminates large errors in both transient and steady-state in comparison to other integral error performance measures such as *ISE, IAE, ITSE*, etc.

2.2. Big bang-big crunch algorithm

The Big Bang-Big Crunch algorithm is basically an optimization method which was proposed by Erol and Eksin in 2006 [8]. This method is similar to GA with respect to generation of initial candidates. The random generation of initial candidates is called *Big Bang phase* (BBP). In this phase, the candidate solutions are disseminated all over the search space in an uniform manner but restricted under search space. The BBP is followed by contraction of solutions in *Big Crunch phase* (BCP) wherein randomly distributed solutions are drawn into order, which can be named as center of mass. Here, the term mass refers to the inverse of the fitness function value. The center of mass $\vec{C}(\vec{x})$ is function of position of each candidate (position vector) in designed search space and is computed as

$$\vec{C}(\vec{x}) = \frac{\sum_{i=1}^{N} \frac{\vec{x}_i}{ISE_i}}{\sum_{i=1}^{N} \frac{1}{ISE_i}}, \tag{1}$$

where $\vec{x}_i$ is i^{th} candidate in n-dimensional search space; ISE_i is treated as an objective function value or fitness function value corresponding to i^{th} candidate, which is defined as

$$ISE_i = \sum_{i=1}^{M} [y(i\Delta t) - \tilde{y}(i\Delta t)]^2, \tag{2}$$