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Configuration and singularity analysis of a parallel hip joint simulator based on the forward kinematics

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ABSTRACT

The singularities of parallel manipulators are usually identified by geometrical methods or by the kinematic principles based on the pose parameters. The methods have limitations in applications that involve singularity avoidance, such as motion planning from input parameters. To identify a singularity from an input parameter point of view, which would make the singularity avoidance strategy more direct and more effective in practical applications, this paper focuses on the relationship between the singularities, the configuration spaces and the input parameters with a 3SPS+1PS parallel hip joint simulator selected to implement this approach. A univariate-form polynomial equation of the forward kinematics is obtained with the aid of the Sylvester dialytic method of elimination, therefore proving that the manipulator has at most eight configurations for a single input. The configurations are then divided into two types of spaces according to their distributions. It is discovered that in practice, we only need concern ourselves with the basic configuration spaces, where the singular loci degenerate into a single surface. Finally, the singular condition is proved to be equivalent to that when the univariate-form input–output equation has a repeated root in the real number field. Therefore, the singular condition equation of the input parameters and the singular loci of the input parameters in the basic configuration spaces are obtained. This study provides new insight into the singularity avoidance of a parallel manipulator, especially for the singularity-free design in the motion planning from input parameters.

1. Introduction

A hip joint simulator using a 3SPS+1PS spatial parallel manipulator as the core module was proposed [1]. This parallel simulator is used to replicate the kinematic and dynamic characteristics of a natural human hip joint to evaluate the friction and wear characteristics of the biomaterials in a hip joint prosthesis. While it overcomes the defects that arise in traditional serial simulators during complex motion simulations with large dynamic loading, it also introduces the inherent defect of all parallel manipulators (i.e., singularities), which results in uncontrolled motion and poor stiffness during certain poses.

The analysis of singularities in mechanisms is a very active research field. A parallel manipulator may either lose or gain one or more degrees of freedom (DOF) in a singular configuration [2], where the Jacobian matrix loses rank, and therefore, it loses the ability to counteract the external forces in certain directions. Researchers have performed a large amount of

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work on the identification and analysis of the singularities in parallel manipulators by means of the Jacobian matrix [3–6], the geometrical method [7–9], the kinematic method [10–14] and screw theory [15–17].

Gosselin [3] proposed a classical way to classify singularities according to the mathematical singularities of two Jacobian matrices, which define the relationship between the input and output velocities. Based on the cascaded expansion of the determinant of the Jacobian matrix, Li [4] presented an analytic form of the six-dimensional singularity locus of the general Gough–Stewart platform. Similar algebraic methods [5,6] would have difficulties dealing with complex mechanisms with multiple DOFs or with special mechanisms.

Merlet [7] introduced the Grassmann geometry for the singularity analysis of a parallel manipulator. The singular configurations were solved using line geometry by looking for possible actuator-line dependencies. This new method was accepted and applied by other researchers [8,9]. However, this method relies heavily on observation, and it is also hard to find all of the singular configurations or to obtain the singularity distribution.

Huang [10] proposed a kinematic method that is based on the relationship of the velocities of any three non-collinear points on the platform. All singularities of the Stewart parallel manipulator were classified into three different linear-complex singularities using this method [11]. Extending this work, Li [12] transformed the singularity analysis into a simpler position analysis of the singularity-equivalent-mechanism and obtained a simpler singularity locus equation. Similarly, Gregorio [13] presented a new expression of the singularity condition for the most general mechanism based on the mixed products of vectors, and he transformed the singularity condition into a ninth-degree polynomial equation that is cubic in the platform position parameters and is a sixth-degree polynomial in the platform orientation parameters. Park [14] first introduced the method of Riemannian geometry for a differential geometric analysis of the kinematic singularities for closed-chain mechanisms. The singularities were classified by configuration space, actuator and end-effector types according to the first-order properties of their kinematics.

Another important tool that has served in the analysis of singularities is screw theory. Hunt [15] laid down the general framework for applying screw theory to singularity analysis, and introduced the notion of stationary and uncertainty configurations. This method is widely used by other researchers, and similar works can be found in [16,17].

Presently, few works have dealt with the relationship between the singularities, the input parameters and the configuration spaces. However, the multiplicity of forward kinematic solutions has been identified as the most significant reason for the singularities that occur in parallel manipulators [18].

Generally, the singularity avoidance is used under three circumstances: (1) online monitoring. In this case, the input parameters are known. However, the pose parameters rely on the detection system, so in this way the singularity avoidance methods based on the input parameters are obviously more direct; (2) motion planning for the pose of a moving platform. In this case, the singularity avoidance methods based on pose parameters are more direct. However, it should be noted that for most parallel manipulators, the inverse kinematics are very simple and have only one solution. Therefore, the input parameters could easily be obtained according to the pose parameters; (3) motion planning for the input parameters (such as returning to zero). Because the forward kinematics of parallel manipulators are extremely complex, and more importantly, the solution is non-unique, which means that the actual output pose is uncertain. This is where those methods based on the pose parameters have limitations whereas the ones based on input parameters would work.

In summary, the singularity avoidance methods that are based on the input parameters would be more direct and more effective than the traditional ones that are based on the pose parameters. Moreover, the analysis of the configuration spaces requires that all possible solutions of the forward position analysis should be found, and the analysis of the singular conditions based on the input parameters also relies on a closed-form model of the forward kinematics. Therefore, an analytic model of the forward kinematics is necessary to conduct the analysis of the relationship between the singularities, the configuration spaces and the input parameters.

This paper is organized as follows. In Section 2, the forward kinematic model is built and verified with a single higher-degree equation obtained. In Section 3, the singular loci of the pose parameters, the distribution of the configuration spaces and their singular status are presented and analyzed. Section 4 gives the singular condition equation for the input parameters and its singular loci in the basic configuration spaces that are more meaningful in practice. Finally, Section 5 follows with the conclusions of this paper.

2. Forward kinematics

The 3SPS+1PS spatial parallel manipulator consists of a moving platform and a base connected by three surrounding SPS-type driving legs and one intermediate PS-type leg, as shown in Fig. 1(a). The driving legs use electric cylinders to drive the moving platform. The intermediate leg is fixed on the base and connected to the center of the moving platform with a thrust bearing. It is used to install the artificial hip joint and balance the loading force of the hydraulic cylinder. The height is determined by the initial adjustments and remains unchanged during the working process. A_i and B_i are the fixed points of the spherical hinges on the moving platform and the base, and they are distributed as two regular triangles, with radiuses e and E, respectively.

The fixed coordinate system $\{\boldsymbol{b}\}$ is established on the base at its center point O with the line OB_2 as the Y-axis and the normal line of the base as the Z-axis. The moving coordinate system $\{\boldsymbol{m}\}$ is established on the moving platform at its center point o with the line oA_2 as the y-axis and the normal line of the moving platform as the z-axis, as shown in Fig. 1(b).