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A new analytic solution for buckling of doubly clamped nano-actuators with integro differential governing equation using Duan–Rach Adomian decomposition method

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ABSTRACT

The Duan–Rach modified Adomian Decomposition Method (ADM) is utilized to obtain a convergent series solution for buckling of nano-actuators subject to different nonlinear forces. To achieve this purpose, a general type of the governing equation for nano-actuators, including integro-differential terms and nonlinear forces, is considered. The adopted governing equation for the nano-actuators is a non-linear fourth-order integro-differential boundary value equation. A new fast convergent parameter, Duan's parameter, is applied to accelerate the convergence rate of the solution. The obtained solution is an explicit polynomial series solution free of any undetermined coefficient. Thus, it can facilitate the design of nano-actuators. As a case study, the results of the approach are compared with the results of Wazwaz Modified Adomian Decomposition Method (MADM) (with undetermined coefficients) as well as a numerical method. The results indicate the remarkable robustness of the present approach.

1. Introduction

A nano/micro actuator is a basic inevitable part of many nano/micro electro mechanical systems. In nano/micro mechanical sensors or actuators, the actuator consists of a beam suspended over a substrate. Applying a voltage difference between the beam and the substrate induces electrostatic field, and as a result, the electrostatic forces attracts the beam into the substrate. The system (i.e. beam and substrate) acts as a capacitor, in which the motion of the beam can be detected by the capacitive change as a signal [1]. The nano-actuators are subject to different inherently nonlinear forces including van der Waals attractions, Casimir force, dielectric effects and fringing field effects [2–4]. The axial forces in the clamped-clamped type of nano-actuators play a crucial role and should be taken into account. The presence of such axial forces adds an integro-differential term to the governing equation of the nano-actuators [2,5–7]. In most of the applications of nano-/micro-actuators, the actuators are manufactured in packs as many as thousands for sensors and billions for chipsets. Hence, developing accurate and fast methods for analysis of nano/micro structures consisting of these actuators is highly demanded.

In many of the recent studies, the modified Adomian Decomposition Method (ADM) with undetermined coefficients were utilized has been utilized to obtain an analytical solution for buckling of the nano-actuators [3,4,8–14]. The results of these

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studies indicate the robustness and capability of ADM to deal with the nonlinearity of the governing equations and provide acceptable accuracy. In the method of undetermined coefficients utilized in [3,4,8–14], there are undetermined coefficients in the solution, which should be determined later using the boundary conditions. Thus, the obtained solution is not in a fully explicit form, and hence, the relations between the affective parameters are not clear. In addition, in many cases, the algebraic equations for evaluating the undetermined coefficients are highly nonlinear and demands numerical procedure. Moreover, in most cases, there are several sets of roots for undetermined coefficients, in which finding and choosing the physical set of roots is a challenge. These are the main disadvantages of ADM which limits the practical application.

Recently, Duan and Rach [15] proposed a new modification of Adomian Decomposition Method (ADM) to deal with the boundary-value problems. They came with the idea that they could use all of the boundary conditions to build a decomposed solution free of any undetermined coefficient. Then, a recursive scheme can be utilized to evaluate the solution in different stages. The method of Duan and Rach is capable of obtaining a fully explicit solution, which is free of any undetermined coefficient. Duan et al. [16] embedded a fast convergent parameter, which can remarkably accelerate the convergence rate of the solution.

In the present study, a general form of the governing equation of nano-actuators, including the integro-differential term is considered. The Duan–Rach modified Adomian decomposition method is utilized to overcome the shortcomings of the conventional Adomian decomposition methods and obtain an explicit analytical solution for buckling of nano-actuators in the presence of different nonlinear effects.

2. Mathematical model

A general form of the governing equation of a nano-actuator beam, including the effect of axial loads and different types of nonlinear forces, in a non-dimensional form can be written as:

$$\frac{d^4u}{dx^4} - \left(\eta\int_0^1\left(\frac{du}{dx}\right)^2 dx + P\right)\frac{d^2u}{dx^2} = -\frac{\alpha}{u^\zeta} - \frac{\beta}{(\kappa+u)^2} - \frac{\gamma}{u}, \tag{1}$$

where u is the non-dimensional deflection of the beam (dependent variable) and x is the non-dimensional length of the beam (independent variable). P, η, β, κ, γ and α are non-dimensional parameters and ζ is an integer positive number. P and η represent the effect of axial forces, β denotes the effect of the external applied voltage, κ represents the effect of a dielectric layer, and γ represents the fringing field or the capillary effect. In the case of $\zeta=3$, α denotes the van der Waals effects, and in the case of $\zeta=4$, α denotes the Casimir effect. It is worth noticing that in the previous studies, some of the non-dimensional parameters of Eq. (1) were neglected. Here, in Eq. (1), we considered all of the terms, which are utilized in different studies, to form a general differential equations. For example, the terms of P in [6], $\eta\int_0^1 (\frac{du}{dx})^2 dx$ in [2,6,7], β in [17,18], γ in [12,13,18], α/u^3 (i.e. $\zeta=3$) in [3,12], α/u^4 (i.e. $\zeta=4$) in [4,10,13] and κ in [2] have been utilized. Neglecting some of these terms reduces the present governing equation to each of the mentioned studies. The clamped-clamped nano-actuators are subject to the following boundary conditions [2,6]:

$$u(0)=1,\quad u'(0)=0,\quad u(1)=1,\quad u'(1)=0. \tag{2}$$

Therefore the goal of the present study is utilizing and modifying the Duan–Rach Adomian method to obtain an explicit convergent series solution for Eq. (1) subject to boundary conditions of Eq. (2).

3. Analytical solution

Here, the modified Adomian decomposition method, proposed by Duan and Rach [15], is applied to obtain a solution for the governing differential equation of the nano-actuator, Eq. (1), subject to the boundary conditions, Eq. (2). Now, starting with the standard ADM, Eq. (1) is written in the operator form as:

$$Lu - \left(\eta\int_0^1 N_2u\,dx + P\right)\frac{d^2u}{dx^2} = N_1u, \tag{3}$$

where $L(.)=d^4/dx^4(.)$ is the linear differential operator to be inverted. N_1u and N_2u are the analytic nonlinear terms where

$$N_1u = -\frac{\alpha}{u^\zeta} - \frac{\beta}{(\kappa+u)^2} - \frac{\gamma}{u}, \tag{4}$$

$$N_2u = \left(\frac{du}{dx}\right)^2. \tag{5}$$

Attention to the boundary conditions shows that the function and its first derivative are given at $x=0$ and $x=1$. Hence, following the Duan–Rach ADM, the inverse operator (L^{-1}) is selected as:

$$L^{-1}(.) = \int_0^x\int_0^x\int_0^x\int_0^x (.)dxdxdxdx. \tag{6}$$