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Fundamental mode of freely decaying laminar swirling flows

F. Beaubert[a], H. Pálsson[b], S. Lalot[a,*], I. Choquet[c], H. Bauduin[a]

[a] TEMPO, UVHC, Campus Mont Houy, 59313 Valenciennes Cedex 9, France
[b] University of Iceland, Sæmundargötu 2, Reykjavík 101, Iceland
[c] University West, Department of Engineering Science, Trollhättan 46186, Sweden

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ABSTRACT

A detailed study of a swirling flow in a tube is presented in the first part of the paper. A simplified analytical solution of the governing equations indicates specific modes of the tangential velocity and that the decay of the swirl effect is exponential. The problem is then solved in three dimensions using computational fluid dynamics (CFD) and a comparison with analytical expressions shows that the CFD code is reliable in terms of accuracy. The CFD results confirm that a fundamental swirling mode is reached within a short distance from the inlet. The torque swirl number is introduced to physically estimate the intensity of the swirl. A companion value is given: it is the average deviation.

1. Introduction

Swirling flows are encountered in many situations, either as a consequential and sometimes unwanted effect or as an induced and controlled effect. Apart from specific utilization of swirling flow that are linked to the increased shear stresses, e.g. [1–3], they are extensively used in burners, e.g. [4–6], for their mixing ability. A specific and important purpose of swirling flow is to increase the convection heat transfer coefficient in tubes. In this context, most of the applications are related to heat exchangers, which is the topic of this paper.

There are many ways to create a swirling flow in tubes. The most widely used technique is based on twisted tapes which can either be continuous or discontinuous, see e.g. [7–9]. These tapes are easily fitted inside tubes, but their main drawback is a quite large increase in the pressure drop along the tubes. When applicability to tube bundles is not an issue, i.e. when space is available around the tube inlet, tangential inlets are often used, where the flow is rotated around the tube axis. More complex geometries are also used, as presented in e.g. [10,11]. A solution similar to the tangential inlet involves installing inserts at the entrance of the tubes, as described in e.g. [12–14]. In one such case, as detailed in [15], it has been shown that the gain in increased heat transfer because of the swirling flow might be higher than the increased pumping energy necessary to overcome the pressure loss due to the swirling flow induction. Thus, an optimal design of heat exchanger tubes can in some cases involve installation of swirling devices in a given set of specific tubes, in place of changing the tube diameters to maximize the heat flow.

* Corresponding author. Tel.: +33 327 511 973; fax: +33 327 511961.
E-mail addresses: francois.beaubert@univ-valenciennes.fr (F. Beaubert), halldorp@hi.is (H. Pálsson), sylvain.lalot@univ-valenciennes.fr (S. Lalot), isabelle.choquet@hv.se (I. Choquet), hadrien.bauduin@univ-valenciennes.fr (H. Bauduin).

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The topic of the present paper is laminar swirling flow in a tube. It is shown in [16] that in the laminar regime the fluid equations representing the swirling flow are linear with respect to the Reynolds number, for a Reynolds number up to 1600. A Reynolds number sastifying this criterion was thus used in this study, $Re = 100$ to be specific. The aim here is to present a semi-analytical solution for the decay of the swirling flow along a tube and compare the results to ones obtained with Computational Fluid Dynamics (CFD) calculations. The overall purpose is to be able to compare with results from CFD calculations in a sufficiently accurate manner using specific inlet profiles for the swirling flow, and consequently ensure the suitability and accuracy of the CFD models in designing and testing optimal swirling flow inlet conditions. Even though most CFD models are at a mature stage with regards to accuracy, especially for laminar flow, an analytical solution with respect to well defined prerequisites is an important tool to ensure the correct physical behavior of CFD calculations and estimate discretization errors always involved in numerical methods. Such analysis results are also of great help in determining appropriate mesh densities in heat exchanger CFD modeling of the type described in the paper.

Most of the studies that have been published until now deal with turbulent flow (see e.g. [17,18]), which is by far the most common flow regime in tubes. Also, some studies have considered flow at lower Reynolds numbers and with regard to such cases a review of swirling effect for transitional flows is given in [19]. However, in the case of tube flow with highly viscous fluids the flow regime can be laminar and in such cases the number of published studies is quite limited, see e.g. [20,21].

The analytical study is presented in the first part of the paper. As a preliminary to the CFD calculations, a new boundary condition is presented in the second part of the paper, developed for the pressure at the outlet of the tube under consideration. The third part is dedicated to the validation of the computations by comparisons with selected analytical solutions. Different inlet profiles are considered and results are analyzed in the fourth part of the paper, with a special attention on the characterization of the swirling effect.

2. Analytical solution

The relations governing steady and incompressible flow are the continuity equation and the momentum equations, relating velocity and pressure, supplemented with appropriate boundary conditions. The geometry studied here is a circular tube and hence it is assumed that the velocity and pressure do not depend on the angular coordinate θ. The reduced set of equations, in cylindrical coordinates (r, θ, z), to be solved is then (the first one represents continuity)

$$\frac{1}{r}\frac{\partial r u_r}{\partial r} + \frac{\partial u_z}{\partial z} = 0 \tag{1}$$

$$\rho\left(u_r\frac{\partial u_r}{\partial r} + u_z\frac{\partial u_r}{\partial z} - \frac{u_\theta^2}{r}\right) + \frac{\partial p}{\partial r} = \mu\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_r}{\partial r}\right) + \frac{\partial^2 u_r}{\partial z^2} - \frac{u_r}{r^2}\right) \tag{2}$$

$$\rho\left(u_r\frac{\partial u_\theta}{\partial r} + u_z\frac{\partial u_\theta}{\partial z} + \frac{u_r u_\theta}{r}\right) = \mu\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_\theta}{\partial r}\right) + \frac{\partial^2 u_\theta}{\partial z^2} - \frac{u_\theta}{r^2}\right) \tag{3}$$

$$\rho\left(u_r\frac{\partial u_z}{\partial r} + u_z\frac{\partial u_z}{\partial z}\right) + \frac{\partial p}{\partial z} = \mu\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_z}{\partial r}\right) + \frac{\partial^2 u_z}{\partial z^2}\right) \tag{4}$$

where $\vec{u}(u_r, u_\theta, u_z)$ denotes the velocity, p the pressure, ρ the density, and μ the dynamic viscosity.

The system of differential equations (Eqs. 1–4) is written in a dimensionless form using $r = r^0 r^*, u_r = u_r^0 u_r^*$, $z = z^0 z^*, u_z = u_z^0 u_z^*$, $u_\theta = u_\theta^0 u_\theta^*$ and $p = p^0 p^*$ where the characteristic variables are indicated with a 0, while the dimensionless variables are indicated with a $*$ symbol. It has to be noted that r^0 and z^0 are of the same order of magnitude. According to [22], the radial velocity of a confined swirling flow is "considerably smaller than the other two components". Then it is possible to write $u_r^0 = \varepsilon_z u_z^0$ and $u_r^0 = \varepsilon_\theta u_\theta^0$ with $\varepsilon_z \ll 1$ and $\varepsilon_\theta \ll 1$. In that case, the equations are written as follows:

$$\varepsilon_z\frac{\partial r^* u_r^*}{\partial r^*} + \frac{\partial u_z^*}{\partial z^*} = 0 \tag{5}$$

$$u_z^*\frac{\partial u_z^*}{\partial z^*} + Eu^0\frac{\partial p^*}{\partial \bar{z}} + \varepsilon_z\left(u_r^*\frac{\partial u_z^*}{\partial r^*}\right) = \frac{1}{Re_r^0}\left(\frac{1}{r^*}\frac{\partial}{\partial r^*}\left(r^*\frac{\partial u_z^*}{\partial r^*}\right) + \frac{\partial^2 u_z^*}{\partial z^{*2}}\right) \tag{6}$$

$$u_z^*\frac{\partial u_\theta^*}{\partial \bar{z}} + \varepsilon_\theta\frac{u_\theta^0}{u_z^0}\left(u_r^*\frac{\partial u_\theta^*}{\partial r^*} + \frac{u_r^* u_\theta^*}{r^*}\right) = \frac{1}{Re_z^0}\left(\frac{1}{r^*}\frac{\partial}{\partial r^*}\left(r^*\frac{\partial u_\theta^*}{\partial r^*}\right) + \frac{\partial^2 u_\theta^*}{\partial z^{*2}} - \frac{u_\theta^{*2}}{r^{*2}}\right) \tag{7}$$

where Eu^0, Re_r^0 and Re_z^0 are the Euler number and Reynolds numbers.