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Laguerre–Galerkin methods with reduced sum-products

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ABSTRACT

The always stable solution of time-domain integral equations by plain implementation of the classical marching-on-in-degree (MOD) methods demands $\mathcal{O}(M^2N^3)$ operations, where M and N are, respectively, the spatial and temporal degrees of freedom. Diverse caching and basis reordering approaches are first explored to reduce the number of matrix–vector multiplications necessary to compute the right hand side of the MOD system. The new schemes primarily eliminate the existing $\mathcal{O}(N)$ innermost vector summations on all past polynomial orders. This complexity reduction facilitates the usage of FFT approaches to accelerate the calculation of existing temporal convolutions. In an additive secondary stage, the recursive time convolution products of the Toeplitz block aggregates of the retarded interaction matrices are expedited by array multiplications in spectral domain. When the translation invariancy on time-order indices are placed on the outermost possible nested Toeplitz levels due to the space shift invariancy, the overall computational complexities and memory requirements can be further reduced to $\mathcal{O}(MN\log(MN))$ and $\mathcal{O}(MN)$, respectively.

1. Introduction

The marching-on-in-time (MOT) schemes are currently the first choice for solving the time-domain integral equations (TDIE) [1]. Much effort has been made to postpone or filtering out the late-time instability in the MOT schemes. Nontheless, among the robust choices of TDIE methods such as the convolution quadrature methods (CQM) [2], the marching-on-in-order, also referred as marching-without-time-variable or marching-on-in-degree (MOD), algorithms are inherently the only TDIE-based solvers that are always stable [3]. The MOT schemes use locally supported (commonly shifted Lagrange) expansion functions and point matching testing in time with interpolation in between past nodal points, while the MOD recipes employ entire-domain and controllably smoother temporal basis functions, i.e. the weighted Laguerre polynomials, together with the Galerkin's method for time testing. In contrast to the non-symplectic MOT methods, the complete set of the Laguerre expansion principally provides a purely energy conserving time integration method, when sufficient number of time bases N are integrated [4]. Hence, the MOD methods substantially offer improved accuracy whereas their computational costs outweigh those of the MOT methods.

The major cost of the MOT, CQM, and MOD approaches are mainly the computation of past solution couplings involving space-time convolution of the induced currents with the retarded Green's function. The computational burden of the classical MOT schemes scales as $\mathcal{O}(N_gNM^2)$, where M denotes the number of subdomain basis functions and N_g is the maximum number of the last retarded time steps till which the scatterer spatial meshes interact. Depending on the electric dimension

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of the scatterer and time step, the terminating delayed samples N_g may vary from $N_g < <N$ to N_g of $\mathcal{O}(N)$. In the CQM and MOD methods, however, independent from the problem size steadily $N_g \geq N$ is held. Generally, large time step sizes, implying small N_g, strengthen the stability of the MOT. On the other hand, the MOD may call for accumulation of plenty of orders N to diminish early ripples in the initial coarse representations of the system response [5]. Moreover, in the MOD, $\mathcal{O}(N)$ inner loop operations are applied on the past current vectors to calculate the integration or differentiation operations. Five marching-without-time-variable recipes, other than the one in [6], are introduced here to omit the yet existing costly $\mathcal{O}(N)$ vector summations on the behind polynomial orders.

As the physics underlying the scattering phenomena is time invariant, when the time axis is discretized uniformly, the retarded TDIE matrices form a Toeplitz system along the temporal steps. The MOT solvers have been boosted by the fast Fourier transform (FFT)-based algorithms [7] in which the convolution products are accelerated by taking advantage of Toeplitz properties in the aggregates of impedance matrices. The time-FFT proposed in [8] does not march in time by the method of lines, rather it solves for all space-time unknowns once (as Rothe's method does), whereas the FFT-accelerated marching solution methods promises the feasibility of long time simulation of large scale scattering problems. The FFT process, however, perishes the sparsity of the MOT matrices, whereas the MOD methods originate dense matrices and hence they demand more than the MOT methods for devising auxiliary techniques to reduce the CPU cycles and memory consumptions.

The usage of spatial FFT for acceleration and compression of single matrix products has been plugged to the MOD recipes [5]. The time-domain adaptive integral method (AIM) [1] uses blocked four-dimensional (4-D) FFTs to connect the implicit temporal arrangement with the 3-D forward spatial FFTs. This implies that the uniform AIM grid has to be zero padded at least to the double size of the auxiliary sources in each dimension to avoid aliasing errors. This paper enables the incorporation of the space and time convolution products of the TDIE methods into single 1-D FFT. It is proven that without considering any spatial property of the matrix blocks [7], the MOD solution cycles also reaches to the logarithmic complexity scaling $\mathcal{O}(N\log^2 N)$ using the Toeplitz property in time dimension. Recently, the subdivision of large-size Toeplitz block aggregates to elementary matrix blocks was proposed to speed up the MOT scheme [9]. The present work shows that for large M, the subdivision to fixed-size blocks alone becomes inferior to the native multilevel aggregate matrix–vector multiply when the MOD in conjunction with mapping to the uniform grid is used. Finally, a hybrid matrix grouping approach is formulated in which the complexity and memory occupation are further reduced by avoiding repeating the FFT computation for current vectors.

2. TDIE and advanced MOD methods

Let S denote the surfaces of perfect electric conducting (PEC) objects that are illuminated by a transient electromagnetic field $\mathbf{E}^i(\mathbf{r}, t)$. The total tangential electric field on S remains zero for ever. As a result, the induced surface current vector $\mathbf{J}(\mathbf{r}, t)$ satisfies the following time-domain electric field integral equation (EFIE):

$$\frac{\mu}{4\pi}\frac{\partial}{\partial t}\int_S \frac{\mathbf{J}(\mathbf{r}',\tau)}{R}dS' - \frac{\nabla_{\mathbf{r}}}{4\pi\epsilon}\int_S\int_{-\infty}^{\tau}\frac{\nabla_{\mathbf{r}'}.\mathbf{J}(\mathbf{r}',t')}{R}dt'dS' = \underline{\mathbf{E}}^i(\mathbf{r},t), \tag{1}$$

where $\underline{\mathbf{E}}^i(\mathbf{r},t) = \hat{\mathbf{n}} \times \left(\hat{\mathbf{n}} \times \mathbf{E}^i(\mathbf{r},t)\right)$, $R = |\mathbf{r} - \mathbf{r}'|$ and the observation point $\mathbf{r}$ and the source point $\mathbf{r}'$ imply arbitrarily located points on the surfaces S. The variable $\tau = t - R/c$ denotes the retarded time, the parameters μ and ϵ indicate the permeability and permittivity of the surrounding environment and $\hat{\mathbf{n}}$ represents an outward-directed unit vector normal to S at field point $\mathbf{r}$.

The derivative counterpart of the EFIE (DEFIE) is also of interest to preferably avoid the laborious computation of charge accumulation in solving the original EFIE (1) by the MOT methods. The DEFIE is obtained by taking a time derivative from both sides of (1):

$$\frac{\mu}{4\pi}\frac{\partial^2}{\partial t^2}\int_S \frac{\mathbf{J}(\mathbf{r}',\tau)}{R}dS' - \frac{\nabla_{\mathbf{r}}}{4\pi\epsilon}\int_S \frac{\nabla_{\mathbf{r}'}.\mathbf{J}(\mathbf{r}',t)}{R}dS' = \frac{\partial\underline{\mathbf{E}}^i(\mathbf{r},t)}{\partial t}. \tag{2}$$

Almost in all MOD studies, the Hertz vector potential has been adapted to solve the EFIE [10–12] instead of direct solving for the unknown surface current. Evidently, the Hertz approaches demand extra post-processing stages for computation of some desired electromagnetic quantities. Excluding the temporal derivation on the excitation term, the Hertz approach results in formulations identical to the DEFIE [11]. Considering S as a closed surface, one may also consider the time-domain magnetic field integral equation (MFIE):

$$\frac{\mathbf{J}(\mathbf{r},t)}{2} - \hat{\mathbf{n}} \times \frac{1}{4\pi}\int_{S_0}\left[\frac{1}{c}\frac{\partial \mathbf{J}(\mathbf{r}',\tau)}{\partial t} \times \frac{\mathbf{R}}{R^2} + \mathbf{J}(\mathbf{r}',\tau) \times \frac{\mathbf{R}}{R^3}\right]dS' = \hat{\mathbf{n}} \times \mathbf{H}^i(\mathbf{r},t_n), \tag{3}$$

where S_0 denotes the surface S from which the contribution of the singularity at $R = 0$ has been removed [13].

This paper aims to accelerate the MOD solution procedures of not only the EFIE (as [6] does), but also the DEFIE and MFIE as well. To numerically solve (1)–(3) or any linear combinations of them, the induced surface current is approximately