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Optimal forced vibration control of a smart plate

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ABSTRACT

In this paper, optimal forced vibration control problem is considered for the rectangular plate with Kelvin–Voigt damping subject to an external excitation with control exercised by piezoelectric patch actuators bonded on both sides of the plate. By deriving the maximum principle, the control problem is reduced to solving a system of partial differential equations for the state variable and the adjoint variable subjected to boundary, initial and terminal conditions. Two cases with respect to forcing function are examined and results are presented.

1. Introduction

Mechanical systems are susceptible to vibrations that might cause some undesirable affects in the structure. Therefore, active vibration control is still one of the appealing topics to the scientists and engineers. With this aim, smart materials are widely used in the active vibration control of structures. For the last two decades, piezoelectric materials are employed especially to damp out the vibrations in the structures due to their large bandwidth, their mechanical simplicity and ability to produce force acting against vibrations in the structures. The general overview of these materials is given in the books [1,2]. The studies about the control of vibration for a plate by means of piezoelectric actuator can be summarized by, but not limited to [3,4].

Particularly, in [4], optimal piezoelectric control of vibrations in a plate is studied where the plate model under consideration does not include the effect of the internal damping due to microstructure of a component contributing to the damping of vibrations modeled as Kelvin–Voigt damping [5]. When a smart material producing Kelvin–Voigt damping is embedded within a host structure, it is essential for engineers to understand the dynamic behaviors of such a system [6]. In [7], it is studied the effect of the material damping to the structures controlled by piezoelectric actuator and it is observed that ignoring the natural damping causes the underestimation of the structural behavior. In this paper, the plate subject to an external disturbance in the presence of the internal damping is studied. Controllability and uniqueness of the control function are also discussed while these concepts are not considered in [4]. Moreover, the systems in [4] and present paper are modeled as self-adjoint and non-self-adjoint systems, respectively. Due to non-self-adjointness of the model used in this paper, the uniqueness of the solution of the state equation is proved elegantly by using energy integral method.

In this study, an active open-loop control is presented to damp out vibrations in a plate with Kelvin–Voigt damping and subject to an external disturbance function in the form of a transverse forcing function. The control is managed by means of piezoelectric patch actuators bonded on the both side of plate and the voltage which to be spent optimally is applied to

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these patches to generate the electrical field to suppress the forced vibrations in the plate over a specified control duration. Due to the presence of the damping term, the governing equation does not admit a variational principle in the classical sense making an approach based on variational calculus not practical. Therefore, the open-loop control law is derived by introducing the adjoint problem and related Hamiltonian in the form of a maximum principle(see, [8–11]). By means of this approach, the problem is reduced to solving a system of partial differential equation. Solution of this system is obtained for using Galerkin expansions of the state and adjoint variables.

The document is organized as follows: The next section is devoted to a presentation of the Mathematical formulation of the control problem. Section 3 and Section 4 are dedicated to introduce the optimal control problem and to the derivation of Maximum principle for present problem, respectively. In Section 5, Galerkin expansion technique is applied to adjoint and state equations to solve the optimal control problem. Finally, to show the validation of the proposed control method, numerical examples are presented by MATLAB.

2. Mathematical formulation of the control problem

Let us consider a three-layer plate which consists of a central host layer and two piezoelectric patch actuators bonded on perfectly to both sides of the plate and these piezoelectric patch actuators and their edges are parallel to the edges of the plate. Also, the piezoelectric actuators on the both sides of the plate are at the same location for the effective control. It is assumed that the plate is subject to the forcing function, initially at rest and undeformed. The aim of the present study is effectively to damp out the vibrations activated by external force by means of a minimum amount of the control applied to patch actuators. Smart structure in two dimension, shown in Fig. 1. with internal damping which is modeled as Kelvin–Voigt damping is given by [12]

$$\eta \bar{w}_{\bar{t}\bar{t}} + \vartheta \nabla^4 (\bar{w} + 2\bar{\xi} \bar{w}_{\bar{t}}) = f + \bar{\mathcal{V}}, \tag{1}$$

where $\bar{\mathcal{V}} = \bar{V}_e(\bar{t}) \nabla^2 \triangle [H(\bar{x}, \bar{y})], \bar{w}(\bar{x}, \bar{y}, \bar{t})$ is deflection function, $x, y \in [0, \ell] \times [0, \ell] = \bar{S}$ are space variables, $(\bar{x}, \bar{y}, \bar{t}) \in \bar{Q} = \bar{S} \times [0, \bar{t}_f]$, $\bar{t} \in [0, \bar{t}_f]$ is time variable, $\bar{t}_f$ is fixed final time, η is a constant, $\bar{\xi}$ is the damping coefficient, $f = f_1(\bar{x}, \bar{y}) f_2(\bar{t})$ is the external force function with f_1 denoting external force dispersion, $\bar{V}_e(\bar{t})$ is the control function and

$$\nabla^2 = \frac{\partial^2}{\partial \bar{x}^2} + \frac{\partial^2}{\partial \bar{y}^2}.$$

Effective flexural stiffness ϑ is defined by

$$\vartheta = \frac{Eh^3}{12(1 - \zeta^2)},$$

in which E is Young's modulus, h is elastic thickness of the plate, ζ is Poisson's ratio. Smart plate equation depends on following

$$\bar{w} = \bar{w}_{xx} = 0 \quad \text{at} \quad x = 0, \ell \tag{2}$$

$$\bar{w} = \bar{w}_{yy} = 0 \quad \text{at} \quad y = 0, \ell \tag{3}$$

and initial conditions

$$\bar{w} = \bar{w}_0 \in H^1(\bar{S}), \quad \bar{w}_{\bar{t}} = \bar{w}_1 \in L^2(\bar{S}) \quad \text{at} \quad t = 0. \tag{4}$$

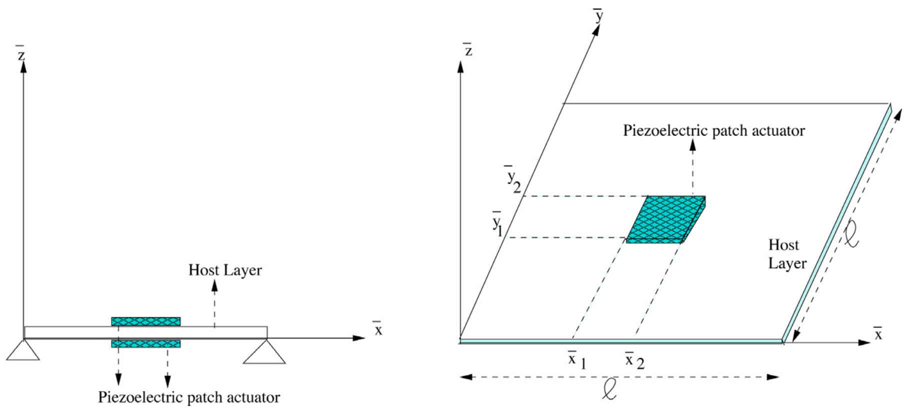

Fig. 1. The cross section (left) and general view (right) of the smart plate.