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Thermomechanical interactions in transversely isotropic magnetothermoelastic medium with vacuum and with and without energy dissipation with combined effects of rotation, vacuum and two temperatures

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ABSTRACT

The present paper is concerned with the investigation of disturbances in a homogeneous transversely isotropic thermoelastic rotating medium with two temperature and magnetic effect due to thermomechanical sources. The formulation is applied to the thermoelasticity theories developed by Green–Naghdi [7] with and without energy dissipation subjected to the thermomechanical sources. Laplace and Fourier transform technique is applied to solve the problem. The bounding surface is subjected to concentrated and distributed sources (mechanical and thermal sources). The expressions of displacement, stress components, temperature change and induced magnetic field are obtained in the transformed domain. Numerical inversion technique has been applied to obtain the results in the physical domain. Numerical simulated results are depicted graphically to show the effect of two temperature and rotation on resulting quantities. Some special cases are also deduced from the present investigation.

1. Introduction

During the past few decades, widespread attention has been given to thermoelasticity theories that admit a finite speed for the propagation of thermal signals. In contrast to the conventional theories based on parabolic-type heat equation, these theories are referred to as generalized theories. Because of the experimental evidence in support of the finiteness of the speed of propagation of a heat wave, generalized thermoelasticity theories are more realistic than conventional thermoelasticity theories in dealing with practical problems involving very short time intervals and high heat fluxes such as those occurring in laser units, energy channels, nuclear reactors, etc. The phenomenon of coupling between the thermomechanical behavior of materials and magnetic behavior of materials has been studied since the 19th century.

Chen and Gurtin [1], Chen et al. [2] and Chen et al. [3] have formulated a theory of heat conduction in deformable bodies which depends upon two distinct temperatures, the conductive temperature φ and the thermo dynamical temperature T. For time independent situations, the difference between these two temperatures is proportional to the heat supply, and in absence of heat supply, the two temperatures are identical. For time dependent problems, the two temperatures are different,

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regardless of the presence of heat supply. The two temperatures T, φ and the strain are found to have representations in the form of a traveling wave plus a response, which occurs instantaneously throughout the body (Boley and Tolins [4]).The wave propagation in two temperature theory of thermoelasticity was investigated by Warren and Chen [5].

Green and Naghdi [6] postulated a new concept in thermoelasticity theories and proposed three models which are subsequently referred to as GN-I, II, and III models. The linearized version of model-I corresponds to classical thermoelastic model (based on Fourier's law). The linearized version of model-II and III permit propagation of thermal waves at finite speed. Green–Naghdi's second model (GN-II), in particular exhibits a feature that is not present in other established thermoelastic models as it does not sustain dissipation of thermal energy [7]. In this model the constitutive equations are derived by starting with the reduced energy equation and by including the thermal displacement gradient among other constitutive variables. Green–Naghdi's third model (GN-III) admits dissipation of energy. In this model the constitutive equations are derived by starting with the reduced energy equation, where the thermal displacement gradient in addition to the temperature gradient, are among the constitutive variables. Green and Naghdi [8] included the derivation of a complete set of governing equations of a linearized version of the theory for homogeneous and isotropic materials in terms of the displacement and temperature fields and a proof of the uniqueness of the solution for the corresponding initial boundary value problem.

A comprehensive work has been done in thermoelasticity theory with and without energy dissipation and thermoelasticity with two temperatures. Youssef [9], constructed a new theory of generalized thermoelasticity by taking into account two-temperature generalized thermoelasticity theory for a homogeneous and isotropic body without energy dissipation. Quintanilla [10] investigated thermoelasticity without energy dissipation of materials with microstructure. Kumar and Devi [11] discussed Magneto thermoelastic with and without energy dissipation Half-Space in contact with Vacuum. Several researchers studied various problems involving two temperature e.g. (Kumar, Sharma and Garg [12]; Kaushal et al. [13]; Kaushal Sharma and Kumar [14]; Kumar and Mukhopdhyay [15]; Ezzat and Awad [16]; Sharma and Marin [17]; Sharma and Bhargav [18]; Sharma, Sharma and Bhargav [19]).

In view of the fact that most of the large bodies like the earth, the moon and other planets have an angular velocity, as well as earth itself behaves like a huge magnet, it is important to study the propagation of thermoelastic waves in a rotating medium under the influence of magnetic field. So, the attempts are being made to study the propagation of finite thermoelastic waves in an infinite elastic medium rotating with angular velocity. Several authors (Das and Kanoria [20]; Kumar and Kansal [21]; Kumar and Rupender [22]; Atwa and Jahangir [23], Mahmoud [24], Sarkar and Lahiri [25]) have studied two-dimensional problem of generalized thermoelasticity to study the effect of rotation.

Inspite of all these investigations, no attempt has been made yet to study the response of thermomechanical sources in transversely isotropic solid with two temperature and magnetic effect and in contact with vacuum in the context of Green Naghdi theories of type-II and type-III. The components of normal displacement, normal stress, tangential stress and conductive temperature subjected to concentrated normal force, uniformly distributed force and Linearly distributed source are obtained by using Laplace and Fourier transforms. Numerical computation is performed by using a numerical inversion technique and the resulting quantities are shown graphically.

2. Basic equations

The simplified Maxwell's linear equation of electrodynamics for a slowly moving and perfectly conducting elastic solid are

$$\text{curl}\,\vec{h} = \vec{J} + \varepsilon_0 \frac{\partial \vec{E}}{\partial t}\ , \tag{1}$$

$$\text{curl}\,\vec{E} \;=\; -\mu_0 \frac{\partial \vec{h}}{\partial t}, \tag{2}$$

$$\vec{E} = -\mu_0 \left(\frac{\partial \vec{u}}{\partial t} \times \overrightarrow{H_0} \right), \tag{3}$$

$$\text{div}\vec{h} = 0. \tag{4}$$

Maxwell stress components are given by

$$T_{ij} = \mu_0 (H_i h_j + H_j h_i - H_k h_k \delta_{ij}), \tag{5}$$

where $\overrightarrow{H_0}-$ the external applied magnetic field intensity vector, $\vec{h}-$ the induced magnetic field vector, $\vec{E}-$ the induced electric field vector, $\vec{J}-$ the current density vector, $\vec{u}-$ is the displacement vector, μ_0 *and* ε_0- the magnetic and electric permeabilities respectively, $T_{ij}-$ the component of Maxwell stress tensor and $\delta_{ij}-$ the Kronecker delta.

The constitutive relations for a transversely isotropic thermoelastic medium are given by

$$t_{ij} = C_{ijkl} e_{kl} - \beta_{ij} T\ . \tag{6}$$

Equation of motion for a transversely isotropic thermoelastic medium rotating uniformly with an angular velocity $\mathbf{\Omega} = \Omega n$, where n is a unit vector representing the direction of axis of rotation and taking into account Lorentz force

$$t_{ij,j} + F_i \;=\; \rho\{\ddot{u}_i + (\mathbf{\Omega} \times (\mathbf{\Omega} \times u))_i + (2\mathbf{\Omega} \times \dot{u})_i. \tag{7}$$