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Sensitivity analysis of correlated inputs: Application to a riveting process model

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ABSTRACT

Sensitivity analysis evaluates how the variations in the model output can be apportioned to variations in model inputs. After several decades of development, sensitivity analysis of independent inputs has been developed very well, with that of correlated inputs receiving increasing attention in recent years. This paper introduces a new sensitivity analysis technique for model with correlated inputs. The new method allows us to quantitatively distinguish the effects of the correlated and uncorrelated variations of the model inputs on the uncertainty in model output. This is achieved by performing covariance decomposition for the uncertainty contribution of the inputs after decoupling the correlated and uncorrelated parts of the component functions in the high dimension model representation (HDMR) of the output. The proposed method can be implemented conveniently with any existing HDMR technique developed for independent inputs without any change of the original algorithm. It can be applied to nonlinear and non-monotonic models with correlated inputs. An additive model, two non-additive models with analytical sensitivity indices, and a riveting process model are employed to test the proposed method.

1. Introduction

Sensitivity analysis evaluates how the variations in the model output can be apportioned to variations in model inputs [1]. After several decades of development, sensitivity analysis of independent inputs has been developed very well [2–6], whereas sensitivity analysis techniques for model with correlated inputs are few in the literature [7–17]. Mckay [7] proposed a replicated Latin hypercube sampling technique (r-LHS) to compute the marginal contribution of the correlated inputs to the response variance, which was applied to the Level E model with correlated inputs in a variance reduction setting [8]. Iman et al. [9] proposed the partial correlation coefficient as a measure of parameter sensitivity for models with correlated inputs based on Latin hypercube sampling [9,10], which was then extended by Xu and Gerner [11] to the random balance design technique [12]. They showed that the extended method outperformed r-LHS in terms of computational cost as it only requires one single sample set. Then, in a second article Xu and Gerner proposed a regression-based method to divide the contribution to the uncertainty in the model output by an individual correlated input into the correlated contribution and the uncorrelated one [13]. This allows identifying the spurious inputs which have an impact on the model output only due to their strong correlations with the other significant ones. However, their method is only suitable for linear models, which is then extended by Hao and Li et al. [14,15] to the nonlinear case.

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The aforementioned sensitivity analysis techniques for correlated inputs consider only the first order contribution of the inputs, whereas interaction often plays an important role in the uncertainty contribution of the inputs. Li et al. [17] proposed a random sampling high dimensional model representation (RS-HDMR) method to divide the contribution by an individual or a set of correlated inputs into correlative contribution and structural one. This allows considering the contribution by the interaction of the correlated inputs to the uncertainty in the model output. Yet, the effectiveness of this method relies on the uniqueness of the HDMR of the correlated inputs which is usually a thorny problem to deal with. Mara and Tarantola [18] derived a set of variance-based sensitivity indices that can measure the marginal and total contributions of an individual correlated input to the output variance. The definition of their sensitivity indices relies on a specific orthogonalisation of the inputs and ANOVA-representations of the model output. Therefore, to estimate all the sensitivity indices of the correlated inputs, one has to test $n!$ (n is the number of the inputs) different orderings of the inputs and estimate the sensitivity indices for each of these orders.

Our objective in this work is to propose a new sensitivity analysis technique for model with correlated inputs, which can separate the correlated and uncorrelated contributions by the inputs to the variance of the model output, including both the individual and interaction contribution of the inputs. This is achieved by decorrelating the component functions of the inputs in the HDMR of the output, and then performing covariance decomposition for the uncertainty contribution of the inputs. We will show that the newly defined sensitivity indices allow correctly identifying the variance contribution of the correlated inputs without specific restriction on the form of the model. Furthermore, their estimation is simple and can be easily achieved by any existing HDMR technique.

The paper is organized as follows. In Section 2 we briefly recall the theory of variance-based sensitivity analysis for model with independent variables. We detail the new method and newly defined sensitivity indices for model with correlated inputs in Section 3. In Section 4, we discuss the computational issue and provide a general procedure for estimating the defined sensitivity indices. An additive model, two non-additive models with analytical sensitivity indices are employed to test the reliability of the proposed method in Section 5. In Section 6, the proposed method is applied to a riveting process model. Section 7 discusses some issues associated with the proposed method, and Section 8 presents conclusions.

2. Review of variance-based sensitivity analysis

Given a model of the form $Y = g(\boldsymbol{X})$, with Y a scalar output and $\boldsymbol{X} = (X_1, X_2, ..., X_n)$ input vector, variance-based sensitivity analysis is closely related to the decomposition of the function $g(\boldsymbol{X})$ itself into terms of increasing dimensionality (i.e. HDMR) [2],

$$g = g_0 + \sum_i g_i + \sum_i \sum_{j>i} g_{ij} + \ldots + g_{12\ldots n} \tag{1}$$

where

$$\begin{aligned}
g_0 &= \int g(\boldsymbol{X}) \prod_{k=1}^{n} f_{X_k}(x_k)\mathrm{d}x_k \\
g_i &= g_i(X_i) = \int g(\boldsymbol{X}) \prod_{k=1,k\neq i}^{n} f_{X_k}(x_k)\mathrm{d}x_k - g_0 \\
g_{ij} &= g_{ij}(X_i, X_j) = \int g(\boldsymbol{X}) \prod_{k=1,k\neq i,j}^{n} f_{X_k}(x_k)\mathrm{d}x_k - g_i(X_i) - g_j(X_j) - g_0 \\
&\ldots
\end{aligned} \tag{2}$$

For the independent inputs, all the component functions in Eq. (1) are mutually orthogonal, and the decomposition is unique which leads to the unique decomposition of the unconditional variance as follows [2],

$$V(Y) = \sum_i V_i + \sum_i \sum_{j>i} V_{ij} + \cdots + V_{12\ldots n} \tag{3}$$

where

$$\begin{aligned}
&V_i = V[g_i] = V[E(Y|X_i)] \\
&V_{ij} = V\left[g_{ij}\right] = V\left[E\left(Y|X_i, X_j\right)\right] - V_i - V_j \\
&\ldots
\end{aligned} \tag{4}$$

Here V_i is the marginal variance (main effect) of X_i which measures the amount of the variance of Y explained by X_i alone, V_{ij} is the cooperative fractional variance of $\{X_i, X_j\}$ that measures the amount of the variance explained by the interaction between X_i and X_j and so on. The total variance of X_i is defined as the sum of all fractional variance containing the factor X_i, i.e.

$$V_i^{\mathrm{T}} = V_i + \sum_{i\in u\subseteq\{1,2,\ldots,n\}} V_u \tag{5}$$